TRAJECTORY OPTIMIZATION OF A BIMODAL NUCLEAR POWERED SPACECRAFT TO MARS

THESIS

Steven Robert Oleson, B.S.

AFIT/GA/ENY/90J-1

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GA/ENY/90J-1

TRAJECTORY OPTIMIZATION OF A BIMODAL NUCLEAR POWERED SPACECRAFT TO MARS

THESIS

Steven Robert Oleson, B.S.

AFIT/GA/ENY/90J-1

Approved for public release

AFIT/GA/ENY/90J-1

TRAJECTORY OPTIMIZATION OF A BIMODAL NUCLEAR POWERED SPACECRAFT TO MARS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Astronautical Engineering

Steven Robert Oleson, B.S.

May, 1990

Acknowledgements

I would like to present my heartfelt thanks for the cooperation and contributions of the following individuals towards my thesis:

Dr. W.E. Wiesel, for his guidance and encouragement as my advisor: his insight was indispensable.

Captain James B. Planeux, USAF, for his advice on advanced propulsion systems and propulsion theory.

The members of the Future Space Systems Branch of the Foreign Technology Division for their understanding and support during my full-time training.

My fiancee, Laura Ann Keneda, for her immense patience, consideration and support, without which this thesis could not have been completed.

Steven Robert Oleson

Table of Contents

List of Figures

Abstract

Minimum flight times for a bimodal, nuclear powered spacecraft are sought. A direct trajectory from Earth to Mars is utilized. Earth escape and Mars braking is accomplished with a high thrust, nuclear thermal propulsion unit, while the interplanetary transit is achieved by a low thrust, electric propulsion unit whose thrusting direction may be varied. An existing method that maximizes circular orbit transfer is adapted to the problem by simplifying the escape and braking conditions and requiring the final orbit to be that of Mars thus obtaining minimum flight times. Low thrust direction history, $\Phi(t)$, excess high thrust fuel division between the escape and braking burns, and optimal escape injection angles are found that determine the minimum flight time. Finally, the size of the low thrust propulsion is also varied to find the minimum time of flight.

CHAPTER I: THESIS OUTLINE

INTRODUCTION:

In 1989, four scientists from the Soviet Union's Kurchatov Institute of Atomic Energy presented a report proposing the use of bimodal nuclear power propulsion system (NPPS) for a manned mission to Mars [1]. The paper, prepared for the Space Power Systems Symposium in Albuquerque, New Mexico, hinted at the use of a nuclear reactor for high thrust propulsion and electric power generation. The report stated that the electric power generated could, in turn, be used to power ion engines thus providing propulsion during the transit from Earth to Mars. The report claimed that the use of such a bimodal system would increase the payload ratio of the Mars spacecraft.

The concept of bimodal nuclear power propulsion is by no means new. During the 1960's, at the height of the nuclear rocket program, several researchers noticed the obvious advantage of utilizing the rocket reactor in a low power output mode when the high thrusting rocket was not in use. The power created was designated for use by the auxiliary systems supporting the spacecraft and crew. However, the merit of creating large amounts of electrical power for electric propulsion was not explored.

PURPOSE, METHOD, ASSUMPTIONS:

The purpose of this thesis is to explore the viability of the bimodal nuclear power propulsion concept from an astrodynamics

point of view. Any advantages of such a system over other conceptual systems using only nuclear or electrical propulsion will be sought. Such advantages may be in reduced time of flight, increased payload ratio or added mission flexibility and spacecraft reusability.

The basic approach will be to model the performance of the spacecraft system using a simplified planar solar system where Earth and Mars have circular, coplanar orbits (the higher order effects of the other planets will be ignored). The high thrust nuclear engine will be assumed to be impulsive and be used for use near the planets (Earth and Mars escape and braking) while the electric drive will be used outside the influences of the planets in 'interplanetary' space. The electric propulsion device will be assumed to be of constant thrust although an optimal thrust vectoring scheme will be sought.

The spacecraft design can be described by separate components (e.g. payload, nuclear engine fuel, nuclear power plant etc.) which will be varied to determine the most advantageous proportions of each. Simplifying factors such as thrust-to-weight capability of the nuclear rocket engine and reactor power production efficiency will be used to allow for various types of engines and reactors to be tested.

The exploration of the bimodal concept will proceed somewhat from the inside out. In Chapter 3, a method of determining optimal electric engine thrust vectoring for minimum time transfer between circular orbits will be recreated using existing work done

by Bryson and Ho [2: p.65-69]. The method will utilize calculus of variations and cost functions to find a set of analytical equations to optimize this two point boundary problem. A numerical program using Haming's integrator and the 'shooting method' (a first order gradient method) will be used to determine the actual results of this nonlinear problem. Chapter 4 will build upon this work by adjusting the end points of the problem to be those of the high thrust escape and braking about Earth and Mars respectively. The injection angle at Earth escape will also be varied. Finally, the spacecraft components will be normalized and appropriate limits upon the performance factors and other parameters will be made.

With this work, completed test cases will be run and the results presented and interpreted. The value of the bimodal power propulsion concept will then be discussed followed by suggestions for follow-on research.

Before starting the solution for optimal electric thrust vectoring, a thorough discussion of the parameters and limitations on a typical manned Mars spacecraft will be presented. Further, a brief explanation of the common trajectories and propulsion techniques that have been proposed and their advantages and disadvantages should be given. These topics are covered in the next chapter.

CHAPTER II MANNED FLIGHT TO MARS

INTRODUCTION:

Although man has dreamt of visiting the planets of the solar system ever since he could see them in the night sky, serious consideration for visiting Mars did not occur until the success of the Apollo Moon landings twenty years ago. Since that time, extensive work on feasible mission scenarios and basic spacecraft designs have been pursued.

Currently, a myriad of methods of manned flight to Mars exist. Each method utilizes specific trajectories and propulsion techniques to achieve the mission. Some examples of trajectories are the Hohmann transfer and the opposition class transfer with Venus fly-by. The spacecraft may be propelled by chemical, nuclear, or electric propulsion, singularly or in some combination. Nuclear propulsion devices and sometimes electric propulsion devices will require a nuclear reactor power plant. For a Mars mission, the solar flux in the latter part of the mission is not sufficient to power an electric propulsion system, consequently, a nuclear electric power source would be required.

The various trajectories and methods of propulsion will be discussed separately. First, however, the effects of interplanetary flight on a human crew (harmful radiation and weightlessness, for example) will be addressed so that the added manned constraints imposed upon the spacecraft design and choice

of trajectory may be considered. The topic of this thesis will be the transportation of a manned spacecraft to a parking orbit around Mars; the logistics and technologies related to getting to and exploring the planet will not be addressed.

MANNED CONSTRAINTS:

The presence of astronauts on a Mars mission adds several constraints upon the spacecraft that could probably otherwise be ignored. Two of the limitations imposed will only be briefly mentioned here. Payload for life support/provisions to support human life in his travels to and from the surface of Mars will greatly reduce the possible automated scientific payload; indeed the requirement that the spacecraft return at all is a great penalty. However, the scientific advantages/disadvantages of sending manned expeditions to Mars is beyond the scope of this review. The other limitation to be briefly mentioned is that of the requirement to limit the human body to only a few g's of acceleration, thus limiting certain propulsion techniques. The relative sizes of the propulsion systems addressed in this paper will, however, never exceed approximately one g.

The two other limitations due to the presence of astronauts are more severe and affect not only the design of the spacecraft and its payload but its flight time: harmful radiations and weightlessness. Both of these phenomena will make it desirable to minimize the time of flight as much as possible.

The presence of harmful radiation during a manned Mars mission is a reality of space travel and should be examined. Several different types of radiation detrimental to humans and electronic components will be present from different sources. Radiation from the Sun consists of high speed (200-400 km/sec) subatomic particles (electrons and protons) often termed 'solar wind'. These normally do not exist in lethal amounts in solar space unless solar flare activity occurs. At such times the crew will require a 'storm shelter' for protection. The solar flare cycle is roughly every eleven years with the next maximum to occur in 1992/3. However, the frequency and severity of these storms is roughly predictable by solar observations of sunspot activity which could give the astronauts up to an hour to prepare for the high radiation influx .

The Earth is shielded from solar radiation by the Van Allen belts which are bands of electrons and protons caught by the magnetosphere of the Earth. While these belts do much to protect the Earth, traversing the highly dense fields is hazardous for any long period of time (more than a few hours).

The galaxy itself is also a source of radiation consisting of cosmic rays. Unfortunately, little is known as to the extent of damage caused by this radiation at this time and shielding against it is thought to be nearly impossible. The final source of harmful radiation could come from a nuclear powerplant and/or propulsion unit. Nuclear reactors give off an abundance of alpha particles, beta particles, gamma rays, X-rays, electrons,

positrons, protons and neutrons all of which are potentially harmful to varying degrees and must be shielded against.

Radiation damage to humans takes two forms: somatic and genetic. Somatic radiation exposure involves the individual while genetic effect involves descendants. Genetic effects usually require a lower threshold than somatic. The basic damage caused to the body by radiation is due to the ionization of the cell molecules. This ionizing energy alters a cell's chemistry which, in turn, impairs or destroys the function of the cell. Radiation in DNA cells can cause mutations in later generations. Sensitive electronics are also susceptible to this damaging ionization energy which can cause either a soft error, (loss of digital information), or a hard error (actual altering of electronic hardware). (see figure 2.1)

For each of the harmful types of radiation different techniques and materials are required to provide adequate protection. Figure 2.2 gives some examples of the required shielding. In some cases, such as the high energy radiation from the Sun, only mass is important in absorbing the energy. For nuclear reactor radiation, on the other hand, neutron attenuation is best achieved by using lithium hydride (LiH), while gamma radiation is best handled by a heavy metal such as tungsten. For further discussion of the radiation effects and shielding requirements consult Angelo and Buden [3: p. 111-131].

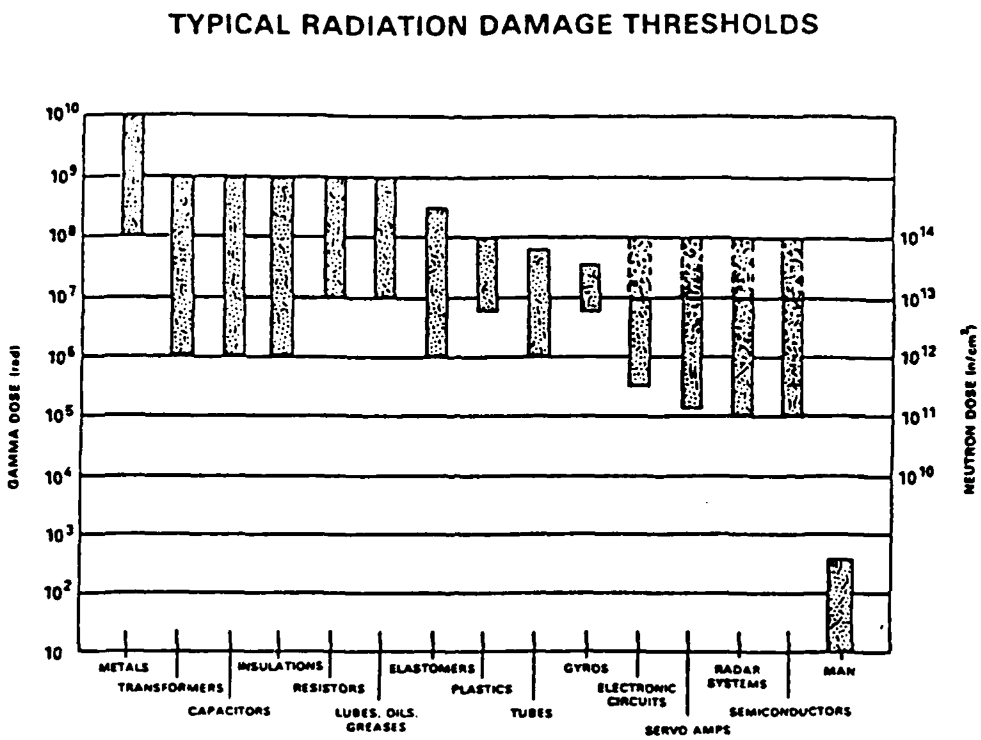


Figure 2.1 Typical radiation damage thresholds [3: p.122]

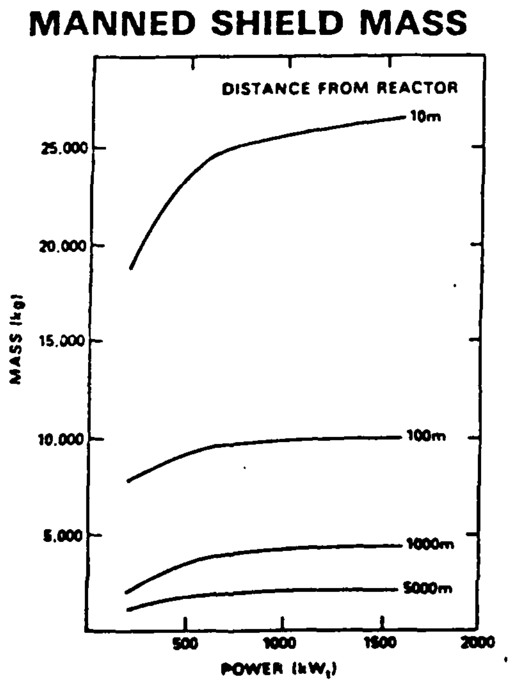


Figure 2.2 Typical 4π shield mass requirements [3: p.129]

The physiological effects of weightlessness also must be considered for the Mars mission. While the initial effects of weightlessness such as motion sickness, dehydration, and muscle deterioration can be countered by appropriate methods, bone deterioration seems to be an irreversible effect. Since the weight bearing bones are no longer affected by gravity, severe calcium loss occurs, seeming to increase linearly with time. Predominantly lost from the arms and legs, the calcium is discharged in the urine. Brittleness of bones and possibly kidney stones would contribute to disable an astronaut for life.

To avert the affects of weightlessness many designers have proposed spinning part or all of the spacecraft to create artificial gravity. Such a requirement would not only greatly complicate the spacecraft design but there is medical concern about the reaction of humans to various speeds of revolution, and the affect of the coriolis force for extended periods of time. In addition, if the astronauts were required to transfer between the artificial gravity conditions and those of weightlessness periodically, motion sickness would occur regularly with unknown results.

In summary, it is evident that substantial protection for the crew from various forms of radiation will be required. For most forms of radiation, it is the length of exposure that dictates the protection required. Excepting artificial gravity, a method of averting bone deterioration has yet to be found. In view of both

these dangers it may be prudent to limit the mission time as much as is possible.

TRAJECTORIES TO MARS:

A multitude of possible transfer orbits exist between Earth and Mars. These may be easily separated into two groups by the general path taken: direct and indirect (conjunction and opposition). Also the way these transfer trajectories may be propelled, by impulsive or low thrust, will affect the transfer that may be achieved. The impulsive trajectory assumes that the propulsion system makes instantaneous changes in velocity or ΔV to instigate a change in orbit. The different types of impulsive and low thrust trajectories will be discussed shortly.

The relative positions of Mars and Earth in the solar system are shown in Figure 2.3. Although both planets seem to have circular orbits about the Sun, on closer inspection it is clear that Earth and Mars have slightly eccentric orbits. Consequently, there exist certain launch windows when the Earth is nearest to Mars, which enable the spacecraft to reach Mars with the least amount of energy required. The Earth is nearest to Mars when Mars is closest to the Sun. These close encounters occur every 15-17 years. However, if the closest proximity requirement is lifted then launch windows (times when the transfer orbit intersects Mars orbit when Mars is at that point) to Mars occur about every 25 months.

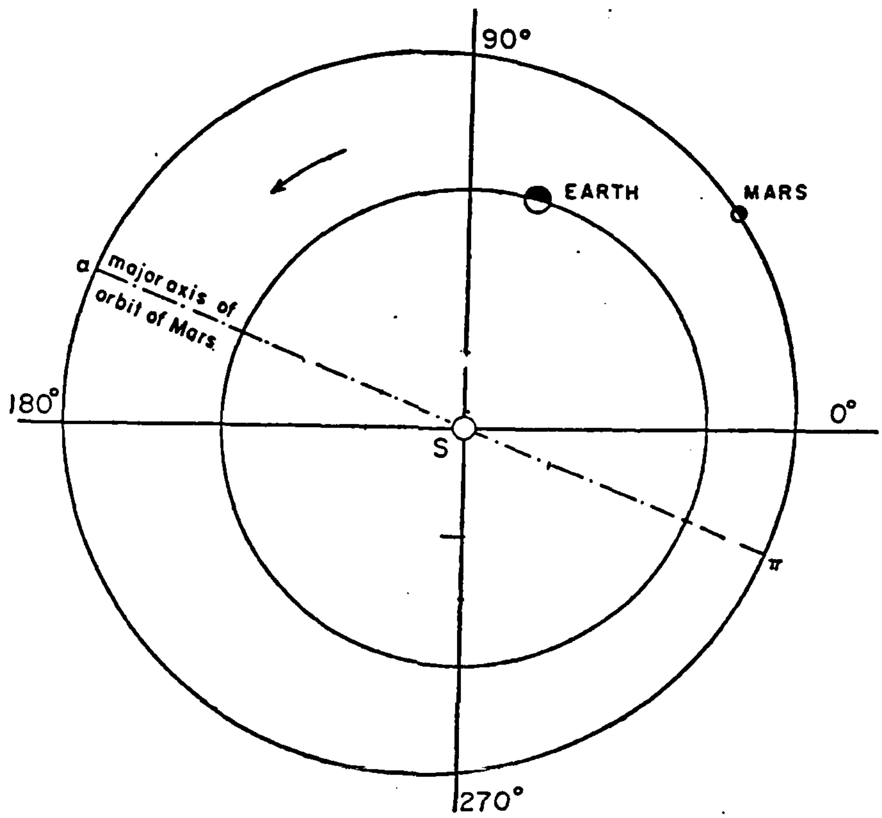


Figure 2.3 Earth-Mars system [4: p.22]

The orbit of Mars is also inclined to that of the Earth. Consequently, any spacecraft travelling between Earth and Mars will be required to perform a plane change of 1.85°. Since the change may be performed at any point in the spacecraft's trajectory it is assumed that it can be done with a minimum of effort and is not usually considered for preliminary mission studies.

The most economical impulsive trajectory of transfer was first discovered in 1925 by the German engineer Hohmann. (see figure 2.4) Applied to an Earth-to-Mars mission the transfer consists of accelerating the spacecraft out of Earth's sphere of influence to the exact velocity which will carry the spacecraft on an elliptical orbit that just reaches the required orbit at aphelion. At this point the spacecraft must make appropriate accelerations to enter the sphere of Mars and park there. This connecting of a planetary hyperbolic orbit with a solar elliptical orbit is termed patched conics from the fact that all impulsive orbits may be geometrically described by a conic, whether an ellipse, parabola or hyperbola.

Using the Hohmann transfer orbit, the spacecraft arrives on the exact point opposite the perihelion point on the other side of the Sun. The Hohmann transfer is consequently just one specific example of a direct or conjugate transfer trajectory and is the lowest energy/longest flight-time of this type. The time the spacecraft may take to directly reach Mars using a Hohmann transfer and an impulsive propulsion device is around 300 days.

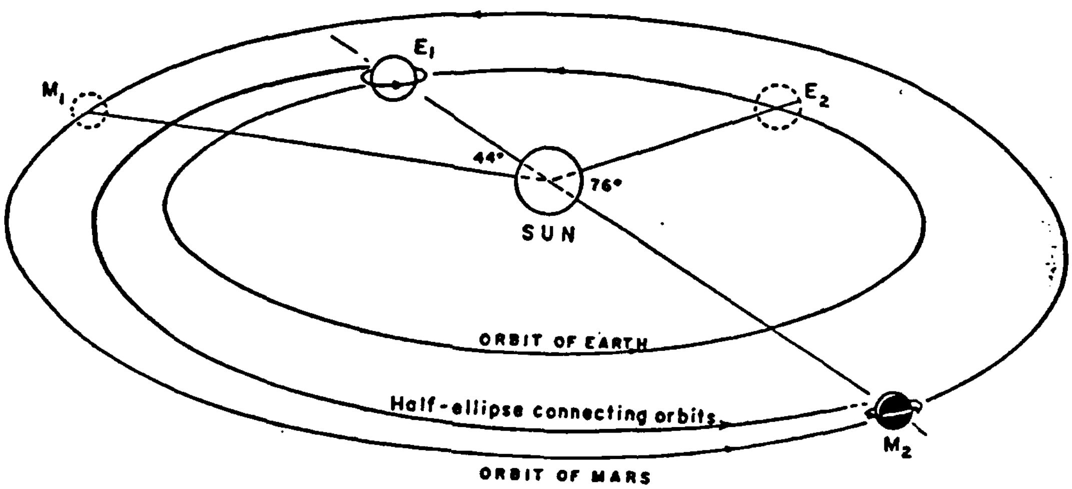


Figure 2.4 Earth-to-Mars Hohmann Transfer [4: p.106]

Unfortunately, the spacecraft must wait at Mars about 400 days before the appropriate return Hohmann ellipse can be used to take them home to Earth; after all, Earth must be at the perihelion of the transfer orbit when the spacecraft arrives there. Consequently, the total mission time would be nearly 1000 days, with about 600 of them spent in interplanetary space transit. Such long periods in space may be detrimental for the crew as discussed previously.

In order to decrease the stay time, the return mission may utilize a high energy indirect or opposition class trajectory. This orbit passes inside the orbit of the Earth and uses another impulsive burn or a close approach to Venus to allow the spacecraft to return to the Earth. (see figure 2.5) While this greatly reduces the stay time to the order of weeks, it requires much more thrusting and will pass much nearer to the Sun than a direct transfer-thus incurring an added shielding requirement. Consequently, a tradeoff exists between adding the provisions and equipment needed for a year stay time or adding the extra shielding and propulsion mass required for an indirect spacecraft.

Regardless of which type of transfer is used, direct or indirect, one may decrease the flight time by allowing for a greater energy allowance. For the direct case, these 'fast' transfer orbits have an aphelion larger than Hohmann's and touch Mars' orbit twice. This trajectory can be performed by impulsively changing the orbit at the first Mars crossing to affect braking into Mars' orbit.

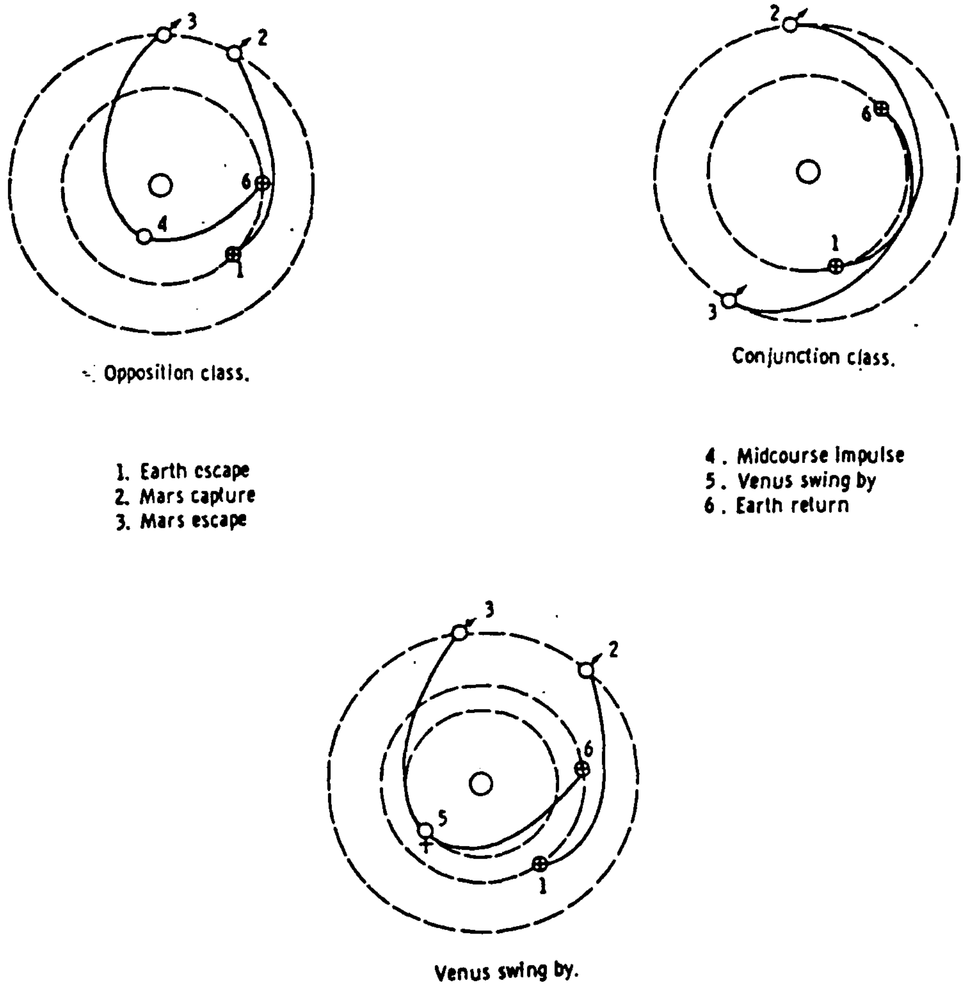


Figure 2.5 Various Earth-Mars trajectories [5]

In general, as discussed by Archer and Cain [5], the performance of the indirect/opposition class is normally marked by shorter mission times and by relatively high energy and severe launch date requirements. Direct/conjunction class missions, while more economical in energy requirements is offset by lengthy total trip times.

For spacecraft utilizing low thrust propulsion units, the trajectory is not defined by a constant conical orbit, rather, the low thrusting is constantly changing the energy and, therefore, the parameters of the orbit. During transit in near planet space, the low thrust driven spacecraft actually has an orbit described by a spiral to affect planetary escape or braking. (see figure 2.6) However, outside the planetary effect, the thrust of the engine is sufficient to provide a more direct trajectory due to the reduced attraction upon the spacecraft. (see figure 2.7) These orbits may also be 'patched' together in an optimal manner as shown by Yoshimura [6].

It is evident that many possible combination of trajectories exist, even combinations of the impulsive (high thrust) and low-thrust types. The trajectory scenario that is chosen depends on many factors-not the least of which are: launch date, allowable transit and stay times, manned constraints, and propulsion system capabilities.

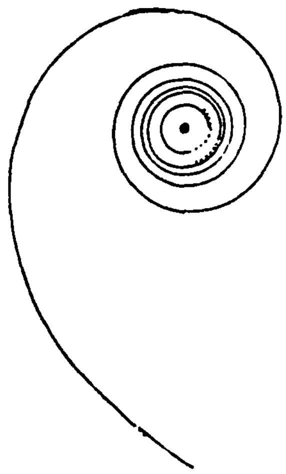


Figure 2.6 Spiral Earth escape [6]

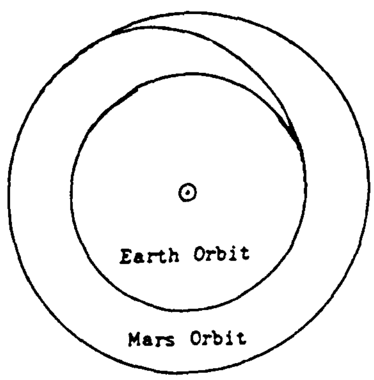


Figure 2.7 Interplanetary low-thrust transfer [6]

PROPULSION SYSTEMS FOR MANNED MARS MISSIONS:

A multitude of manned missions to Mars have been envisaged in the last thirty five years. Each plan utilizes various trajectories and propulsion systems to fulfill mission constraints. The main categories of feasible propulsion technologies available to such a mission are chemical, nuclear, and electric. While chemical propulsion is the only technique used proficiently today, we shall see that its low specific impulse makes it much less attractive than the use of nuclear or electric propulsion.

In 1956, Ley and von Braun [4] proposed a manned mission to Mars utilizing chemical propulsion and a Hohmann transfer. The mission would use two spacecraft, one for passengers and one for the landing craft. Only the passenger spacecraft would return to Earth. Chemical engines using hydrazine fuel and nitric acid oxidizer would be used. The specific impulse of such a system is around 220 sec. The parameters and performance of this mission, as well as the others to be discussed, are given in Figure 2.8. Due to the low performance of the propulsion system, the mass ratio of the spacecraft at Mars to that at the Earth parking orbit is only 0.09. Consequently, ninety percent of the original spacecraft was either made up of propellant or was discarded before Mars capture. This value shows that once the mass of the manned support systems and landers designated to reach Mars has been decided upon, the propulsion system (discarded chemical stages + propellant) will be about nine times this mass which

FIGURE 2.8 COMPARISON OF MARS MISSION SCENARIOS

MISSION CONCEPT:	EARLY CHEMICAL	ALL ELECTRIC	NUCLEAR / ELECTRIC
PROPOSED BY:	LEY / VON BRAUN	KING / STUHLINGER	KING / STUHLINGER
YEAR OF PROPOSAL:	1956	1962	1966
SPACECRAFT:			
# PASSENGER SHIPS:	1	2: RETURN FUEL	3 - 4
# CARGO SHIPS:	2	3: LANDERS	---
INITIAL MASS(TON)/MAN	3400	120	174
MISSION TIMES (DAYS)			
TOTAL TRIP TIME:	969	560	476
EARTH SPIRAL ESCAPE:	---	48	---
EARTH-TO-MARS TRANSIT:	260	140	145
MARS SPIRAL ARRIVAL:	---	21	23.5
STAY TIME:	449	14	30
MARS SPIRAL ESCAPE:	---	21	17.5
MARS-TO-EARTH TRANSIT:	260	268	255
EARTH SPIRAL ARRIVAL:	---	48	5
TRAJECTORY TYPE:	HOHMANN	DIRECT/INDIRECT	DIRECT/INDIRECT
PROPULSION SYSTEM:	CHEMICAL	ELECTRICAL	STAGE-1 NCLR/ELEC.
FUEL:	HYDRAZINE	CESIUM	CRYO. H2 / MERCURY
OXIDIZER:	NITRIC ACID	N/A	N/A
Isp: (sec)	220	14,300	800 / 8000
SPECIFIC PWR (KW/KG):	N/A	0.5	0.16
FINAL MASS RATIO (MARS):	0.09	0.83	0.44
FINAL MASS RATIO:	0.02	0.50	0.25
INITIAL EARTH ORBIT (KM)	8108	540	485
MARS PARKING ORBIT (KM)	1000	300	1000
FINAL EARTH ORBIT (KM)	90000	540	30,000

equates to a large launch vehicle requirement for minimal payoff. This mission uses a Hohmann transfer and requires nearly a 1000 day mission with over 400 days of that spent in interplanetary space under potentially harmful radiation and weightlessness. Therefore, this mission scenario is very unattractive.

It is obvious that the payload ratio must be improved to create a more cost effective system. To improve the useful payload ratio while still utilizing chemical propulsion, aerobraking at Mars' and Earth's atmospheres has been proposed. These aerobraking maneuvers would greatly reduce the amount of propellant required for braking at Mars and the return to Earth. Such techniques are possible but require a much more complex spacecraft design. In addition, the Mars aerobraking would be even more risky than the one at Earth due to our lack of knowledge about the dynamics of Mars' atmosphere.

For either chemically-based scenario, the possibility of utilizing a reusable spacecraft design is almost nonexistent due to the extremely low payload ratios. Consequently, a repeat mission would require a entirely new spacecraft and propulsion system, the latter part being the dominant. By increasing the payload ratio it may be possible to reuse the spacecraft and the propulsion system and only launch the required fuel for a new mission.

Such an alternative to these one time manned Mars missions would be a reusable spacecraft utilizing propulsion concepts more

suitable to interplanetary travel. Since the required length of flight time to Mars is on the order of hundreds of days and the spacecraft is not required to produce a thrust to weight ratio greater than one as are Earth launch vehicles, higher specific impulse and lower thrust systems may be used. Two such techniques that have been explored extensively are nuclear rocket engines and electrical rocket drives. Appendix A contains a review of each of these propulsion concepts.

In the early 1960's, several design studies of electrically propelled manned Mars vehicles were completed. For example, Stuhlinger and King [8: p. 338-343] proposed a manned Mars spacecraft using an all electric propulsion system as described by Figure 2.8. The concept consisted of five spacecraft, three of which carry Mars landers, the other two carrying return fuel for all the spacecraft. The flight times are less than that of an impulsive Hohmann Earth-to-Mars leg of the flight, but an opposition return is used to reduce stay times to a reasonable length. The required fuel for the flight to Mars is only seventeen percent of the original mass of the fueled spacecraft. On return to Earth, the spacecraft mass would be half that of the original and this includes the discarded Mars lander. Such final mass ratios would allow for more reasonable Earth based launch requirements and perhaps spacecraft reusability. Unfortunately, the mission would be hazardous for the crew; long term effects of 400 days of weightlessness are as yet barely known, and radiation exposure for the crew, especially during the month-long escape from Earth through the Van Allen radiation belts would have to be

countered by adequate shielding at a definite mass penalty.

A more recent proposal by Stuhlinger [9: p. 288-301] fitted a nuclear powered rocket stage on the spacecraft to replace the slow spiraling escape from Earth. The added weight penalty of the nuclear stage is evident from the lower mass ratios in Figure 2.8, but the long spiraling out to escape Earth and the slow passage through the Van Allen belts would be avoided. The Nerva II first stage would be discarded after thrusting and the trip would be completed with electric propulsion alone. While the specific power capability of this spacecraft is more realistic than the previous example the reasonable final mass ratios are somewhat misleading since the spacecraft will be returning to a high Earth circular orbit to avoid slow passage through the Van Allen belts. This would strand the crew until rescue from Earth or a low orbiting space station. Reusability of the spacecraft would also be questionable.

From these mission scenarios, some conclusions are evident: nuclear and electric missions merit serious consideration since they far out perform conventional chemical missions, exclusively electric missions deliver the best payload ratios but expose the crew to long periods of radiation and weightlessness (including spiral transit through the Van Allen belts), and combined nuclear/electric propelled missions avoid slow planetary escape/arriving spirals and long term exposure to Earth's Van Allen Belts. With these conclusions in mind, perhaps a manned mission to Mars would be best performed by two different types of

spacecraft. A fleet of unmanned cargo ships using only electrical propulsion could carry all the necessary equipment and landing craft necessary for the manned landing. In addition, the cargo ships could even carry the return propellant necessary for the faster travelling passenger ship. This passenger ship could use either nuclear or nuclear/electric propulsion to propel the manned crew to Mars as quickly as required. Thus, the threats of radiation and weightlessness would be reduced and the payload mass could be used to exclusively support the manned personnel for interplanetary travel.

As mentioned in chapter 1 the purpose for this report will be to determine in what combination nuclear and electric propulsion would be advantageous for the passenger spacecraft, if any.

CHAPTER III: OPTIMAL THRUST VECTORING FOR A MAXIMUM CIRCULAR ORBIT TRANSFER

INTRODUCTION:

Given a spacecraft which utilizes a low thrust propulsion system (i.e. ion drive) providing some constant thrust T, operating for a given flight time, t_f, it is desired to find the optimal thrust-direction history $\phi(t)$, which transfers the space craft from an initial circular orbit, r_o, to the maximum circular orbit, r_f, as defined in Figure 3.1 . This problem has been solved in various publications, most notably by Bryson and Ho [2: p. 65-69]. The analytical formulation and its numerical solution will be presented here for the case of transfer from Earth's solar orbit to Mars' solar orbit since this basic exercise will be modified in the next chapter. Later in this chapter, the adaption of an outer loop to drive the final maximum r_f to be the solar radius of Mars and, consequently, find the minimum time of flight will be presented. Before discussion of the optimization methodology, a brief derivation of the equations of motion will be made.

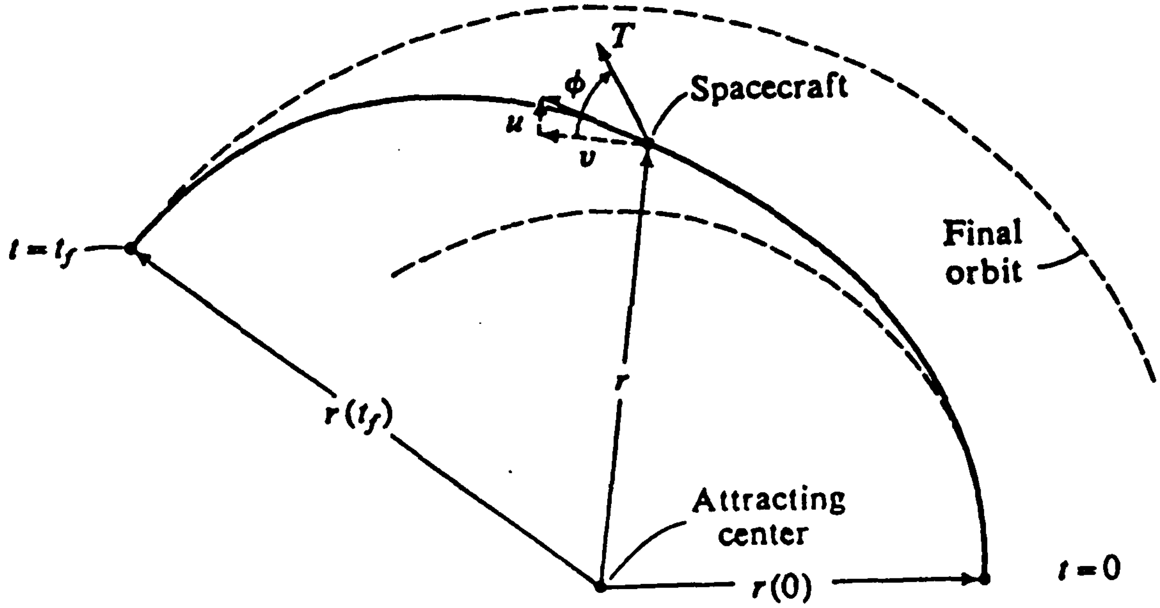


Figure 3.1 Definition of system parameters [2: p.66]

EQUATIONS OF MOTION:

The two-body problem is represented by Figure 3.1 and the nomenclature defined as follows:

r = radial distance of spacecraft from attracting center

u = radial component of velocity

v = tangential component of velocity

m_o = initial mass of spacecraft

$\mathring{m}$ = fuel expulsion rate (constant)

ϕ = thrust direction angle

μ = gravitational constant of body at attracting center

θ = angular position of spacecraft

If the theory of Lagrange's equations of motion are used:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k \qquad k = 1, 2, \ldots n$$

n differential equations

and $L = T_e - V_p$

where

Q_k: nonconservative forces

q_k: generalized coordinates

V_p: potential energy of the system

T_e: kinetic energy of the system

L : Lagrangian of the system.

The generalized coordinates of the polar form are chosen as r and θ. The only nonpotential force acting on the system is the space craft's thrust which may be stated in the radial and tangential components as

$$Q_r = T \sin \phi \quad \text{and} \quad Q_t = r\, T \cos \phi ,$$

where ϕ is the thrusting control angle. In this case the kinetic energy is

$$T_e = \frac{1}{2} (m) \; (v^2 + u^2) ,$$

where m = instantaneous mass. By noting that $v = r\dot{\theta}$ and $u = \dot{r}$ this may be simplified to

$$T_e = \frac{1}{2} (m) \; (r^2 \dot{\theta}^2 + \dot{r}^2).$$

The potential energy is (using Newton's inverse square law)

$$Vp = - m \frac{\mu}{r} ,$$

where μ is the gravitational coefficient of the attracting body. The resulting Lagrangian is

$$L = T_e - V_p = m \left(\frac{1}{2} (r^2\dot{\theta}^2 + \dot{r}^2) + \frac{\mu}{r} \right) .$$

Applying Lagrange's equation to each generalized coordinate:

For $q = r$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = Q_r$$

$$\dot{u} - \frac{v^2}{r} + \frac{\mu}{r^2} = \frac{T \sin \phi}{m} .$$

For $q = \theta$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = Q_t$$

$$r^2 \ddot{\theta} + 2r\dot{r}\dot{\theta} = r \frac{T \cos \phi}{m} .$$

By noting that $\ddot{\theta} = \dfrac{\mathring{v}}{r} - \dfrac{u\ v}{r^2}$, this may be transformed into

$$\mathring{v} + \frac{u\ v}{r} = \frac{T \cos \phi}{m} .$$

Thus, the resulting equations of motion are

$$(1) \quad \mathring{r} = u$$

$$(2) \quad \mathring{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + \frac{T \sin \phi}{m}$$

$$(3) \quad \mathring{v} = -\frac{u\ v}{r} + \frac{T \cos \phi}{m} .$$

It should be mentioned here that an equation for the change in θ is not necessary since it will be assumed that the spacecraft's departure from Earth's solar orbit will be phased correctly so that it will arrive at the proper point in orbit to intercept Mars in its orbit.

DERIVATION OF ANALYTICAL OPTIMIZATION EQUATIONS:

The problem will be solved using a cost function and corresponding analytical equations for a continuous system where *functions* of state variables are specified at fixed terminal time, as described by Bryson and Ho. By following the example in Bryson and Ho, the maximum radius orbit transfer in a given time may be determined. By creating a simple outer loop to vary the time of flight the aforementioned optimization may repeatedly maximize the radius until the radius is equivalent to Mars' orbit. Although

utilization of an outer loop to minimize time to Mars' orbit may take slightly longer than an algorithm to directly minimize time, the outer loop method seems to provide flexibility and better convergence. (The direct method was attempted but had poor convergence qualities, perhaps due to an error in application.) The following is a summary of the equations of formulation.

The cost function may be defined as

$$J = E\,[x(t_f),t_f] + \int_{t_o}^{t_f} L[x(t),u(t),t]\,dt.$$

This cost function value J is to be extremalized to solve the problem. By adjoining the constraints

$$\psi\,[x(t_f),t_f) = 0 \quad (\psi \text{ a } q \text{ vector function}) ,$$

and the system of differential equations

$$\dot{x} = f[x(t),u(t),t] \quad (\text{with } t_o \text{ given}) ,$$

to the performance index J with Lagrange multipliers ν and $\lambda(t)$, the more specific cost function becomes:

$$J = [\varepsilon + \nu^T\psi]_{t=t_f} + \int_{t_o}^{t_f} \{L(x,u,t) + \lambda^T[\,f(x,u,t) - \dot{x}\,]\}\,dt$$

For the cost function to have a stationary value (minimum or maximum) certain necessary conditions are required. These conditions include boundary conditions and side constraints specific to each problem. For a thorough derivation of these equations see Bryson and Ho [2: p.42-69]. The necessary conditions may be summarized with the other equations as follows:

$$\dot{x} = f(x,u,t) \qquad \text{(n differential equations)}$$

$$\dot{\lambda} = -\left(\frac{\partial f}{\partial x}\right)^T \lambda - \left(\frac{\partial L}{\partial x}\right)^T \qquad \text{(n differential equations)}$$

$$\left(\frac{\partial H}{\partial u}\right)^T = \left(\frac{\partial f}{\partial u}\right)^T \lambda + \left(\frac{\partial L}{\partial u}\right)^T = 0 \qquad \text{(m algebraic equations)}$$

$$X_k(t_o) \text{ given, or } \lambda_k(t_o) = 0 \quad (k = 1,\ldots,n) \qquad \text{(initial boundary conditions)}$$

$$\lambda(t_f) = \left(\frac{\partial \varepsilon}{\partial x} + \nu^T \frac{\partial \psi}{\partial x}\right)^T_{t=t_f} \qquad \text{(n boundary conditions)}$$

$$\psi\,[x(t_f), t_f] = 0 \qquad \text{(q side conditions)}$$

These equations and conditions will now be applied to our specific problem. The state vector is assumed to be

$$X = \begin{bmatrix} r \\ u \\ v \\ \lambda_r \\ \lambda_u \\ \lambda_v \end{bmatrix}, \qquad \text{where } \lambda\text{'s are lagrange multipliers.}$$

The equations of motion are (as defined previously)

$$\dot{r} = u\,,$$

$$\dot{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + a(t)\sin\phi\,,$$

and $$\dot{v} = -\frac{uv}{r} + a(t)\cos\phi.$$

The acceleration supplied by the propulsion system is defined as $a(t) = \dfrac{T}{m_o - \dot{m}\, t}$. It is evident that the value of a(t) will increase due to the decrease in the mass of the spacecraft as described by the fuels mass flow rate. The Hamiltonian of the system is defined as

$$H = L(x,u,t) + \lambda^T(t)\ f(x,u,t)$$

Since the function to be maximized contains only end conditions,it is assumed that $L \equiv 0$. Consequently the Hamiltonian of the system in question is

$$H = \lambda_r u + \lambda_u \left(\frac{v^2}{r} - \frac{\mu}{r^2} + \ a(t) \sin\ \phi \right) + \lambda_v \left(-\frac{uv}{r} \ + a(t) \cos\ \phi \right).$$

Since the final radius is to be maximized we wish to set $\Phi = r(tf)$ and ,in turn since $E = \varepsilon + \nu^T \psi$ then

$$E = r(t_f) + \nu_1\ u(t_f) + \nu_2\ \left(v(t_f) - \sqrt{\frac{\mu_\odot}{r(t_f)}}\ \right),$$

where our necessary conditions requiring a maximized final circular orbit have been applied. As such, the final radial velocity should be zero and the tangential velocity equal to that of a circular orbit at the maximum $r(t_f)$. The Lagrangian differential equations defined by

$$\dot{\lambda} = - \left(\frac{\partial f}{\partial x}\right)^T \lambda\ ,$$

are as follows:

$$\dot{\lambda}_r = - \lambda_u \left(- \frac{v^2}{r^2} + \frac{2\ \mu}{r^3} \right) - \lambda_v \left(\frac{uv}{r^2}\right)$$

$$\dot{\lambda}_u = - \lambda_r + \lambda_v \left(\frac{v}{r}\right)$$

$$\dot{\lambda}_v = -\lambda_u \left(\frac{2v}{r}\right) + \lambda_v \left(\frac{u}{r}\right) .$$

These are necessary conditions to find the optimum in an allowable region. The control law is derived from

$$\left(\frac{\partial H}{\partial u}\right)^T = \left(\frac{\partial f}{\partial u}\right)^T \lambda + \left(\frac{\partial L}{\partial u}\right)^T = 0.$$

In this system u(t) the control parameter (not the radial velocity) is merely the ion thrusting angle $\phi(t)$. Applying, the equation to our problem we get

$$(\lambda_u \cos \phi - \lambda_v \sin \phi)\, a(t) = 0.$$

To drive this equation to zero either part of the equation may be zero. Since our spacecraft requires thrust the second term may not be zero, $a(t) \neq 0$. Thus the first term must become zero as

$$\lambda_u \cos \phi - \lambda_v \sin \phi = 0.$$

This is the desired control constraint. This interprets easily to $\tan(\phi) = \frac{\lambda u}{\lambda v}$ or $\phi = \tan^{-1} \frac{\lambda u}{\lambda v}$. This function may be substituted into the differential equations using

$$\sin(\phi) = \frac{\lambda_u}{\sqrt{\lambda_u^{\,2} + \lambda_v^{\,2}}}$$

and

$$\cos(\phi) = \frac{\lambda_v}{\sqrt{\lambda_u^{\,2} + \lambda_v^{\,2}}} .$$

The boundary conditions of our problem consist of initial and final conditions which have either desired values or may be varied freely to achieve the desired optimization.

The initial conditions of the state variables r, v, and u are chosen for an initial circular orbit. In this example the spacecraft will be assumed to be in a circular orbit approximating Earth's solar orbit.

The initial r, u, and v conditions are thus

$$r(0) = r_\oplus \qquad \text{(Earth's radial distance from Sun)} ,$$

$$u(0) = 0 \qquad \text{(radial component)} ,$$

$$v(0) = \sqrt{\frac{\mu_\odot}{r_\oplus}} \qquad \text{(tangential component)} .$$

Since the entire state is defined, the initial Lagrange conditions are free to enable the optimization of the problem:

$$\lambda_r(0) = \text{free} \qquad \lambda_u(0) = \text{free} \qquad \lambda_v(0) = \text{free}.$$

The final conditions Ψ_1, and Ψ_2 constrain the final orbit to be circular:

$$\Psi_1 = u(t_f) = 0 ,$$

$$\Psi_2 = v(t_f) - \sqrt{\frac{\mu_\odot}{r(t_f)}} = 0.$$

The boundary conditions involving the Lagrange multipliers, λ_k, are defined by

$$\lambda(t_f) = \left(\frac{\partial \varepsilon}{\partial x} + \nu^T \frac{\partial \psi}{\partial x}\right)^T_{t=t_f} \quad \text{(n boundary conditions)} .$$

Applying this to the problem, we find:

$$\lambda_r\ (t_f) = 1 + \frac{\nu_2 \sqrt{\mu_\odot}}{2\ [r(t_f)]^{3/2}}\ ,$$

$$\lambda_u(t_f) = \nu_1,$$

$$\lambda_v(t_f) = \nu_2\ .$$

Hence, $\lambda_r(t_f)$ defines the third final constraint,

$$\Psi_3 = \lambda_r(t_f) - 1 - \frac{\nu_2 \sqrt{\mu_\odot}}{2\ [r(t_f)]^{3/2}} = 0.$$

IN SUMMARY:

STATE VECTOR

$$X = \begin{bmatrix} r \\ u \\ v \\ \lambda r \\ \lambda u \\ \lambda v \end{bmatrix}$$

DIFFERENTIAL EQUATIONS

$$f = \dot{X}$$

EQUATIONS OF MOTION

$$\dot{r} = u$$

$$\dot{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + a(t)\ \frac{\lambda_u}{\sqrt{\lambda_u{}^2 + \lambda_v{}^2}}$$

$$\dot{v} = -\frac{uv}{r} + a(t)\ \frac{\lambda_v}{\sqrt{\lambda_u{}^2 + \lambda_v{}^2}}$$

where $a(t) = \dfrac{T}{m_o - \mathring{m}(t)}$.

LAGRANGE MULTIPLIER EQUATIONS

$$\dot{\lambda}_r = -\lambda_u \left(-\frac{v^2}{r^2} + \frac{2\mu}{r^3}\right) - \lambda_v \left(\frac{uv}{r^2}\right)$$

$$\dot{\lambda}_u = -\lambda_r + \lambda_v \left(\frac{v}{r}\right)$$

$$\dot{\lambda}_v = -\lambda_u \left(\frac{2v}{r}\right) + \lambda_v \left(\frac{u}{r}\right)$$

CONTROL LAW

$$\tan(\phi) = \frac{\lambda u}{\lambda v}$$

INITIAL CONDITIONS

$r(0) = r_{\oplus}$ (Earth's radial distance from Sun)

$u(0) = 0$ (radial component)

$v(0) = \sqrt{\frac{\mu_{\odot}}{r_{\oplus}}}$ (tangential component)

$\lambda_r(0) = \text{free}$ $\lambda_u(0) = \text{free}$ $\lambda_v(0) = \text{free}$

FINAL CONDITIONS

$$\Psi_1 = u(t_f) = 0$$

$$\Psi_2 = v(t_f) - \sqrt{\frac{\mu_\odot}{r(t_f)}} = 0,$$

$$\Psi_3 = \lambda_r(t_f) - 1 - \frac{\nu_2 \sqrt{\mu_\odot}}{2\,[r(t_f)]^{3/2}} = 0,$$

$$\lambda_u(t_f) = \nu_1 = \text{free}, \qquad \lambda_v(t_f) = \nu_2 = \text{free}.$$

NUMERICAL SOLUTION:

The equations of motion and optimization derived above are nonlinear ordinary differential equations and consequently must be solved numerically [10]. In this case the equations of motion will be integrated using a fourth order predictor - corrector method created by Haming and thus called Haming's Integrator. This integrator uses a fourth order polynomial and four data points to predict the next point in the integration. Although Haming's method is very quick by other integrator standards,it requires an initialization method to create four points at the beginning of the integration.

By using the Haming Integrator as a fortran subroutine with another subroutine to calculate the specific equations of motion and an appropriate main program to control the subroutines, the equations of motion may be integrated for a certain time period,

t_o to t_f. The results of the integration are unique to the initial conditions; trajectories 'close' to this unique trajectory are not known. Such trajectories are needed if an initial guessed trajectory is to be used to eventually determine the optimal trajectory.

To find these 'close' trajectories the equations of variation are found. For the given system equations of motions,

$$\mathring{x} = f(x,t), \qquad \text{(N differential equations)}$$

a general nearby trajectory may be presented as

$$x = x + \delta x.$$

By substituting this equation into the system equations of motion and expanding in a Taylor's series about $\delta x = 0$ the following equation is evident:

$$\delta\mathring{x} = \left.\frac{\partial f}{\partial x}\right|_{x_o} \delta x.$$

These are the equations of variation. The partial derivative of the vector f (equations of motion) with respect to the vector x (state vector) will be a square matrix abbreviated $A(t) = \partial f/\partial x$. Since the equations of variation are linear, the summation of any solution functions is also a solution and a general solution may be constructed from a fundamental set of solutions. This set may be written in square matrix form denoted by $\Phi(t,t_o)$. This matrix then satisfies

$$\mathring{\Phi}(t,t_o) = A(t)\ \Phi(t,t_o),$$

with initial conditions $\Phi(t_o,t_o) = I$. (where I is the identity matrix.) Consequently, the Φ matrix (called the *state transition matrix*) will propagate the initial deviation from the x_o trajectory to any time in the integration,

$$\delta \mathbf{x}(t) = \Phi(t,t_o)\ \delta \mathbf{x}(t_o).$$

Thus the N^2 equations of variation may be integrated in parallel with the N equations of motion so that all aspects of trajectories 'close' to the nominal trajectory will be known. For this problem, the state vector and differential equations have been derived previously. Using these the **A** matrix is:

$$\mathbf{A}(t) =$$

$$\begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -(v/r)^2+2\mu/r^3 & 0.0 & 2v/r & 0.0 & -Q\lambda_u{}^2+TF & -Q\lambda_u\lambda_v \\ uv/r^2 & -v/r & -u/r & 0.0 & -Q\lambda_u\lambda_v & -Q\lambda_v^2 + TF \\ \lambda_u(-2v^2/r^3+6\mu/r^4)+2\lambda_v uv/r^3 & -v\lambda_v/r^2 & (2v\lambda_u-u\lambda_v)/r^2 & 0.0 & v^2/r^2-2\mu/r^3 & -uv/r^2 \\ -\lambda_v/r & 0.0 & \lambda_v/r & -1.0 & 0.0 & v/r \\ (2v\lambda_u-u\lambda_v)/r^2 & \lambda_v/r & -2\lambda_u/r & 0.0 & -2v/r & u/r \end{bmatrix}$$

$$\text{where}\quad TF = \frac{a(t)}{\sqrt{\lambda_u{}^2+\lambda_v{}^2}} \quad\text{and}\quad Q = \frac{a(t)}{(\lambda_u{}^2+\lambda_v{}^2)^{1.5}}$$

Now that trajectories close to the nominal are available, a way of applying the optimizing equations is needed. In this case the system is a two point boundary problem. The method to be used

to solve the problem is often termed the 'shooting method'. Basically, a first guess is made and the trajectory is propagated to the prescribed final time. At this point the error vector G is calculated as G = (desired final state) - (actual final state) or $G = [\ \Psi_1,\ \Psi_2,\ \Psi_3]^T$. (Which are merely the final conditions, which we desire to be zero.) By adjusting G with the appropriate $\delta x(t_2)$ the condition may be driven to zero. In other words, it is desired to adjust the final conditions so that:

$$G\ [\ x(t_2) + \delta x(t_2)\ ] = 0.$$

By a simple Taylor expansion:

$$G[\ x(t\ )] + \left.\frac{\delta G}{\delta x}\right|_{t_2} \delta x(t_2) \cong 0.$$

By definition, the state transition matrix $\Phi(t_2,t_1)$ is the linearization of the actual transition of the initial deviation from the final deviation:

$$\delta x(t_2) = \Phi(t_2,t_1)\ \delta x(t_1).$$

By substitution and defining $B = \frac{\delta G}{\delta x}$

$$G[x(t\ )] + B\ \Phi(t_2,t_1)\ \delta x(t_1) = 0,$$

which, in turn, may be inverted to find the linearized correction to the initial conditions to reduce the error:

$$\delta x(t_1) = -\ (B\ \Phi(t_2,t_1))^{-1}\ G[x(t\ )].$$

For this problem the error vector is $G = [\Psi_1, \Psi_2, \Psi_3]^T$, where

$$\Psi_1 = u(t_f)$$

$$\Psi_2 = v(t_f) - \sqrt{\frac{\mu_\odot}{r(t_f)}}$$

$$\Psi_3 = \lambda_r(t_f) - 1 - \frac{\nu_2 \sqrt{\mu_\odot}}{2\,[r(t_f)]^{3/2}}$$

Therefore, the $B = \delta G/\delta x$ matrix is

$$\begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \dfrac{\sqrt{\mu}}{2\ r^{1.5}} & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ \dfrac{3\ \lambda_v \sqrt{\mu}}{4\ r^{2.5}} & 0.0 & 0.0 & 1.0 & 0.0 & \dfrac{-\sqrt{\mu}}{2\ r^{1.5}} \end{bmatrix}$$

Consequently, once B and $\Phi(t_2,t_1)$ are determined, multiplied, and inverted with the error vector, the improved corrections, $\delta\lambda^T$, are added to the variable initial conditions (in this case the three initial Lagrange multipliers):

$$\lambda^T_{new} = \lambda^T_{initial} + \delta\lambda^T(t_1).$$

(Note that the initial parameters r_o, u_o, and v_o are assumed for the specified optimization and are not to be modified.)

This procedure is iterated until the error vector reaches an acceptable minimal amount or diverges to infinity. The latter case indicates that a better initial guess that is 'close' to the actual optimal initial conditions is required. A flowchart describing the method is shown in Figure 3.2.

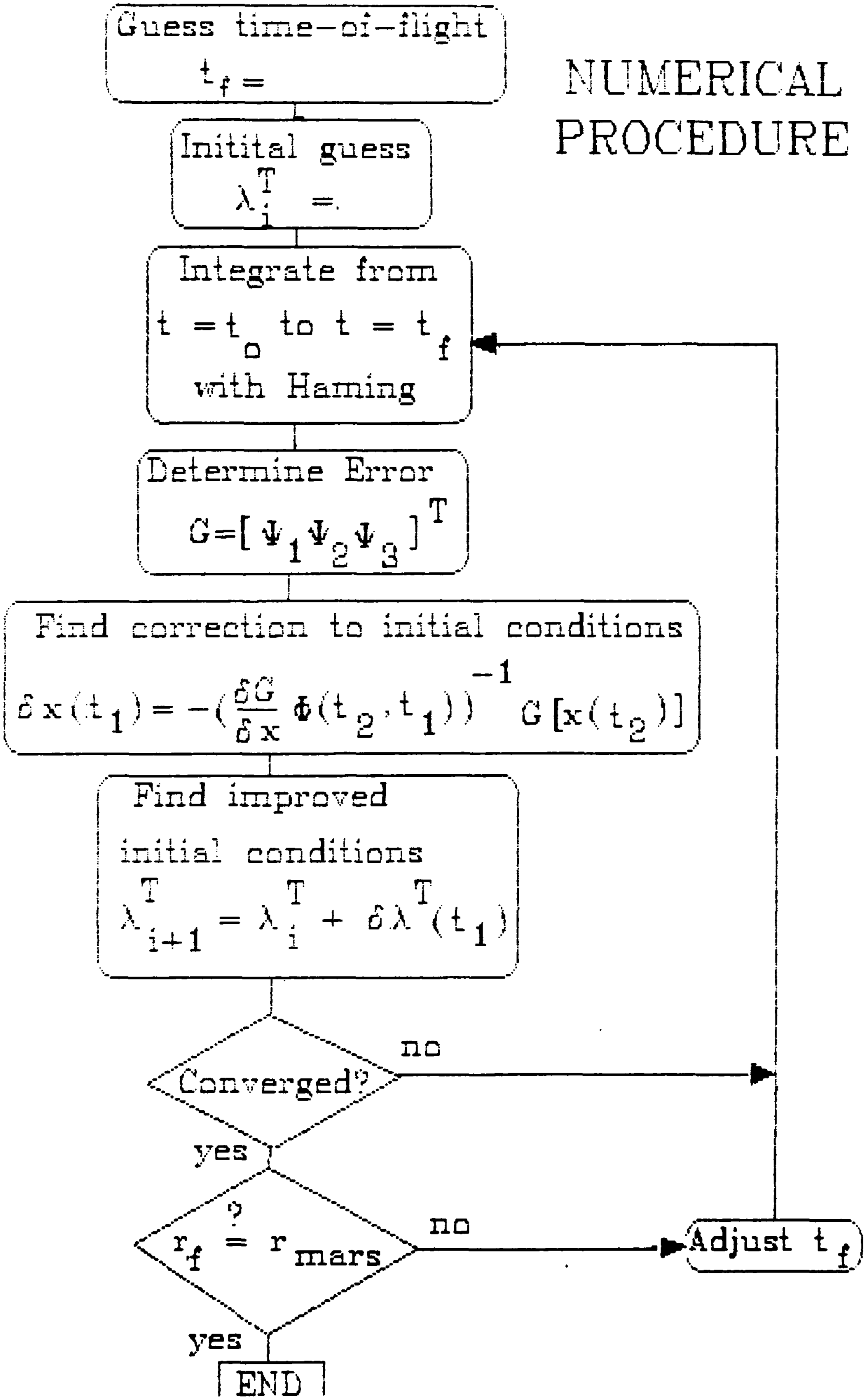


Figure 3.2 Flowchart of numerical procedure

ADJUSTING FOR MINIMUM TIME OF FLIGHT:

The above procedure will maximize the final circular orbital radius for the given time of flight t_f. To adjust this to find the minimum time to reach a certain circular orbit, in this case Mars' solar orbit, a simple outer loop may be written using a scalar Newton-Raphson procedure to adjust the time of flight as

$$t_{f_{k+1}} = t_{f_k} + \left(\frac{t_{f_k} - t_{f_{k-1}}}{r(t_{f_k}) - r(t_{f_{k-1}})}\right)[r_f - r(t_{f_k})].$$

This method has been described by Moyer and Pinkham [11]. Initially the previous point size is assumed since the method requires two points to function. With this modification, the program will find the minimum time of flight transfer from Earth's solar orbit to Mars' solar orbit (neglecting eccentricities) by varying the constant thrust vector. This outer loop is also shown by the flowchart in Figure 3.2.

RESULTS:

By using the same normalized spacecraft parameters assumed in Moyer and Pinkham:

$r_o = 1.000$	$v_f = .8098$
$r_f = 1.525$	$u_f = 0.000$
$\mu = 1.000$	$m_o = 1.000$
$v_o = 1.000$	$\dot{m} = -.07487$
$u_o = 0.000$	$T = .1405,$

the previous algorithm was programmed using FORTRAN.

The results agreed very well with those given by Bryson and Ho as shown in Figures 3.3 and 3.4. The spacecraft's thrust vector is shown to be roughly 60 degrees for the first half of the flight and -60 degrees for the second half of the flight. The thrust vectoring in relation to the orbital transfer is shown in Figure 3.5.

Now that the problem of optimal thrust vectoring has been repeated, the previous algorithm and analytic derivation may be modified for the flight of a bimodal nuclear propelled spacecraft from an Earth parking orbit to a Mars parking orbit. This is the topic of the following section.

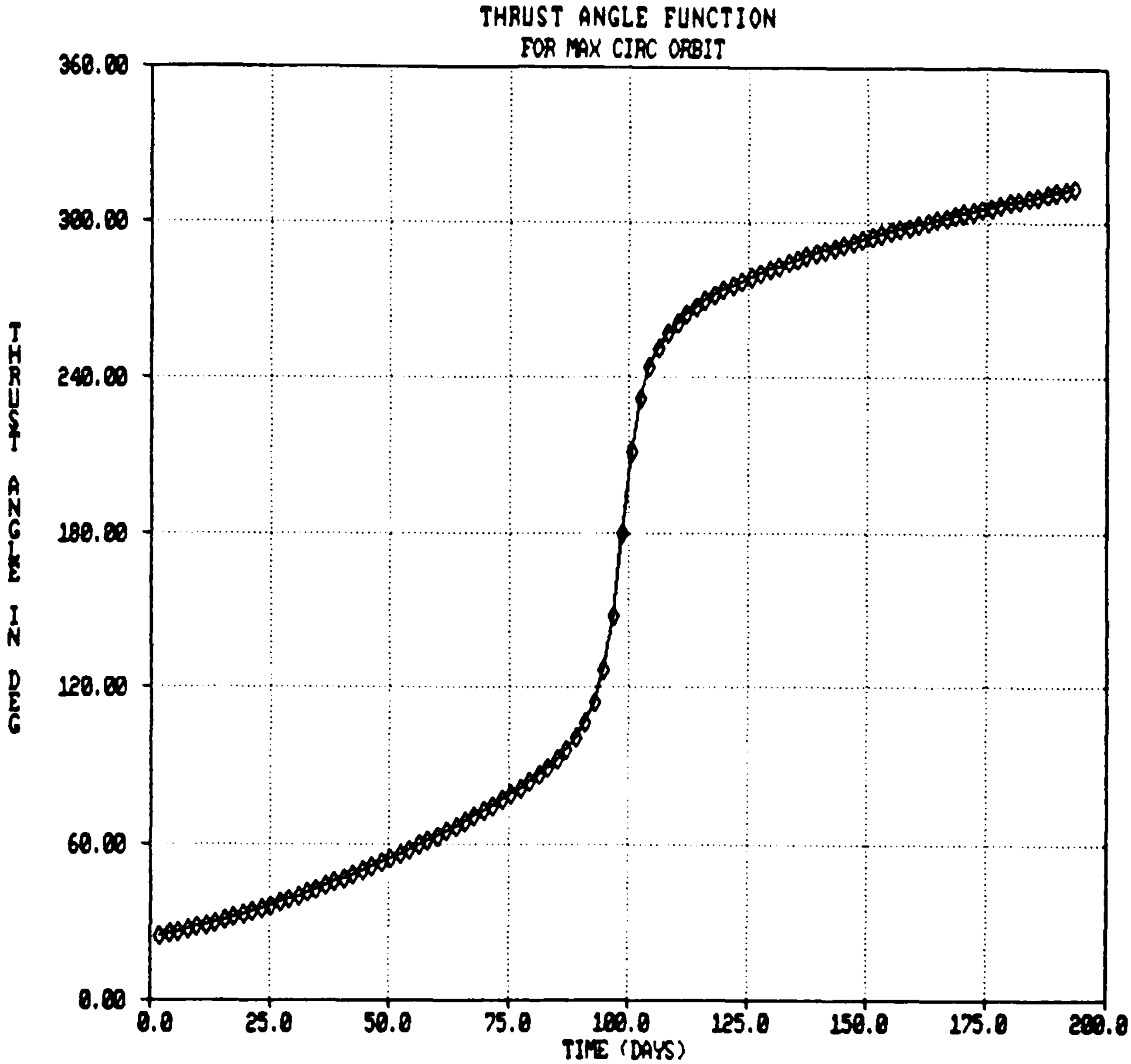


Figure 3.3 Optimal thrust vectoring

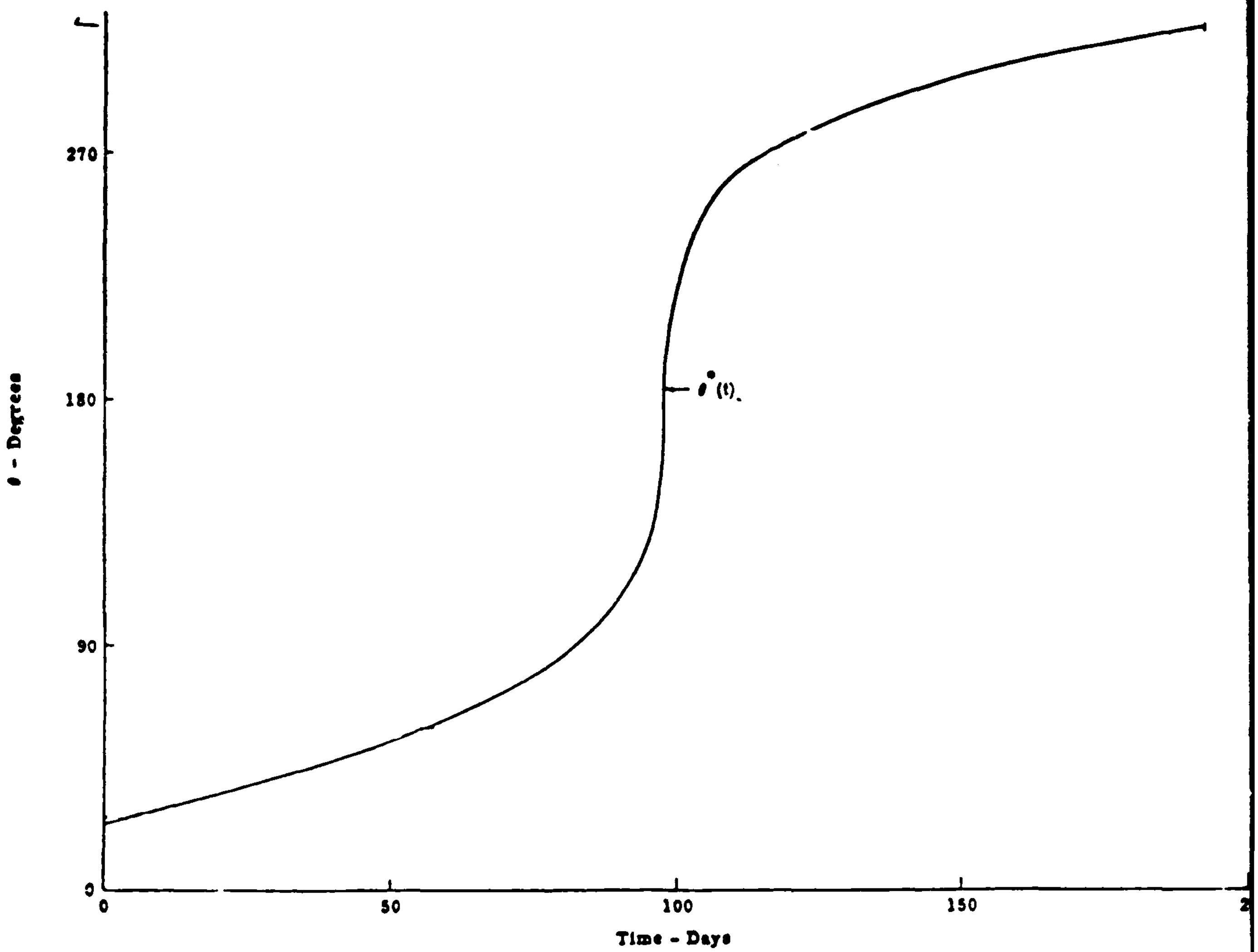


Figure 3.4 Optimal thrust vectoring

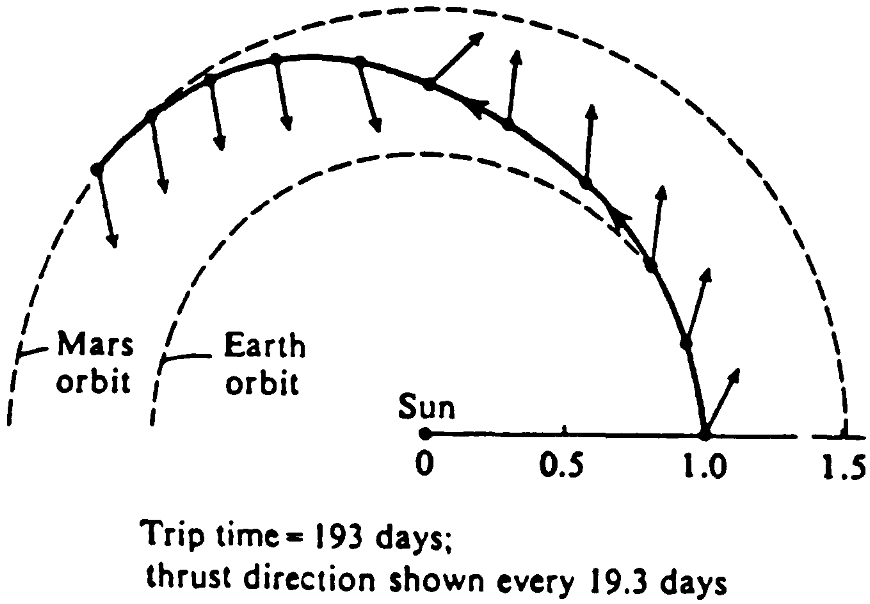


Figure 3.5 Thrust direction during trajectory [2: p.68]

CHAPTER IV: TRAJECTORY OPTIMIZATION FOR A BIMODALLY PROPELLED MANNED MARS MISSION

INTRODUCTION:

The optimization of thrust vectoring derived and completed in the last chapter will now be modified to include the Earth escape and the Mars capture. The Earth-to-Mars spacecraft will utilize an impulsive thrust (instantaneous velocity change from the nuclear rocket) for planetary escape/capture and a non-impulsive, constant thrust (i.e. ion drive) propulsion capability for the interplanetary transfer. The use of high thrust impulsive propulsion will remove the long spiraling times required for escape or capture at a planet as discussed in chapter II.

PURPOSE:

It is desired to find the minimum transfer time while allowing for varied excess impulsive ΔV (i.e. excess impulsive fuel beyond that needed for an escape from Earth and braking at Mars.) This total impulsive ΔV_{tot} is divided into a ΔV_1 and a ΔV_2 which represent the Earth escape required impulse and the Mars capture impulse, respectively. To simplify this minimization, the optimization will instead focus on maximizing the final orbital radius and then finding the time to make this radius equivalent to Mars' solar orbit as was done in the previous section. Once this optimization performs correctly, the amount of low thrust capability for the spacecraft will be varied to determine the advantage of a bimodal nuclear power propulsion system. Before

discussing the method of optimization, several simplifying assumptions will be made.

ASSUMPTIONS AND NOMENCLATURE:

Several simplifying assumptions concerning the solar system can be made. The escape from Earth's sphere of influence to that of the Sun is assumed instantaneous. This seems reasonable since this distance is less than one percent of the mean radial distance from Earth to Mars. Additionally, average escape times are less than seven days while the total mission time (one way) is around two hundred days. Using the same reasoning, the arrival into Mars' sphere of influence is also assumed instantaneous. The orbits of Earth and Mars are assumed to be in the same plane and perfectly circular. This simplifies problem setup. The intercept of the spacecraft with Mars will be assumed. No phasing requirements will be considered.

Certain assumptions for the vehicle must also be made. The ion engines will operate at constant thrust and only outside of the Earth's and Mars' spheres of influence. However, the ion thrust will have variable vectoring or thrust angle Φ equivalent to that defined in the last chapter. Consequently, $\Phi(t)$ will be found to optimize the system. The high thrust engines will be assumed to provide an instantaneous (impulsive) ΔV. The other parameters of the propulsion systems will be normalized in a later section.

PROBLEM NOMENCLATURE AND FORMULATION:

The nomenclature of the problem is as follows :

r = *radial distance of spacecraft from Sun*

u = *radial component of velocity*

v = *tangential component of velocity*

m_o = *initial mass of spacecraft*

$\mathring{m}$ = *ion fuel expulsion rate (constant)*

ϕ = *thrust direction angle*

μ = *gravitational constant of Sun*

ΔV_{tot} = *total velocity change possible for the impulsive engines*

ΔV_1 = *velocity change for Earth escape*

ΔV_2 = *velocity change for Mars capture*

$\Delta V_{tot} = \Delta V_1 + \Delta V_2$

ψ = *injection angle, defines point in parking orbit when impulsive burn begins*

ν_∞ = *arc traversed during change from parking orbit to sphere of influence*

$V_\oplus$ = *orbital velocity of the Earth*

V_{mars} = *orbital velocity of Mars.*

ANALYTIC FORMULATION:

The problem will be solved using a cost function and corresponding analytical equations for a continuous system where functions of state variables are specified at a fixed terminal time, as described in the previous chapter. The equations of formulation for this problem are the same as for that of the last chapter; only the boundary conditions will be different.

These boundary conditions will now allow for a non-tangential escape/arrival with respect to each planet. The sphere of influence was defined by Laplace as the radial distance centered about the smaller of two celestial objects (Earth) that divides the regions where the body in question (i.e. spacecraft) is assumed totally under the influence of only one of the large bodies (the planet or the Sun). This assumption assumption of instantaneous escape from the sphere of influence is accurate enough for mission planning and preliminary trajectory design. This radial distance is shown by

$$\text{Sphere Radius} = D \left[\frac{\text{mass of planet}}{\text{mass of Sun}}\right]^{0.4},$$

where D is the distance between the planet and Sun. The sizes of Earth's and Mars' sphere of influence are 924119 km and 577724 km respectively. These distances account for only 1.2 % and 0.7% of the distance between the planets.

Normally, the escape hyperbola is 'patched' to an elliptical transit orbit in interplanetary space. In this case the components of velocity at the edge of the planet's sphere of influence will be used as the initial/final conditions. These boundary conditions have either required values or may be varied freely to achieve the desired optimization.

INITIAL CONDITIONS:

The initial conditions of the state variables r, u, and v are derived from the initial impulsive parameters ΔV_1 and ψ as referred to in Figure 4.1. Both of these values define the r,u,and v parameters at the escape from the Earth, which is our actual assumed starting point since the time for the escape from the Earth's sphere of influence (SOI) is neglected. The derivation of r,v,and u follows.

The radius of an orbit r is defined by

$$r = \frac{a\ (1-e\ )^2}{1 + \ e\cos\nu}$$

from the general equation of a conic section. Since $r \approx \infty$ at the Earths sphere of influence (let the subscript $_\infty$ denote values at the SOI boundary)

$$r \ = \infty = \frac{a\ (1-e\ )}{1 + e\cos\nu_\infty}$$

consequently, the denominator must be zero, $1 + e\cos\nu_\infty = 0$, so $\cos\nu_\infty = -\frac{1}{e}$. By definition, the eccentricity is

$$e = \sqrt{1 + \frac{2\ \mathcal{E}\ h^2}{\mu_e^2}}\ ,$$

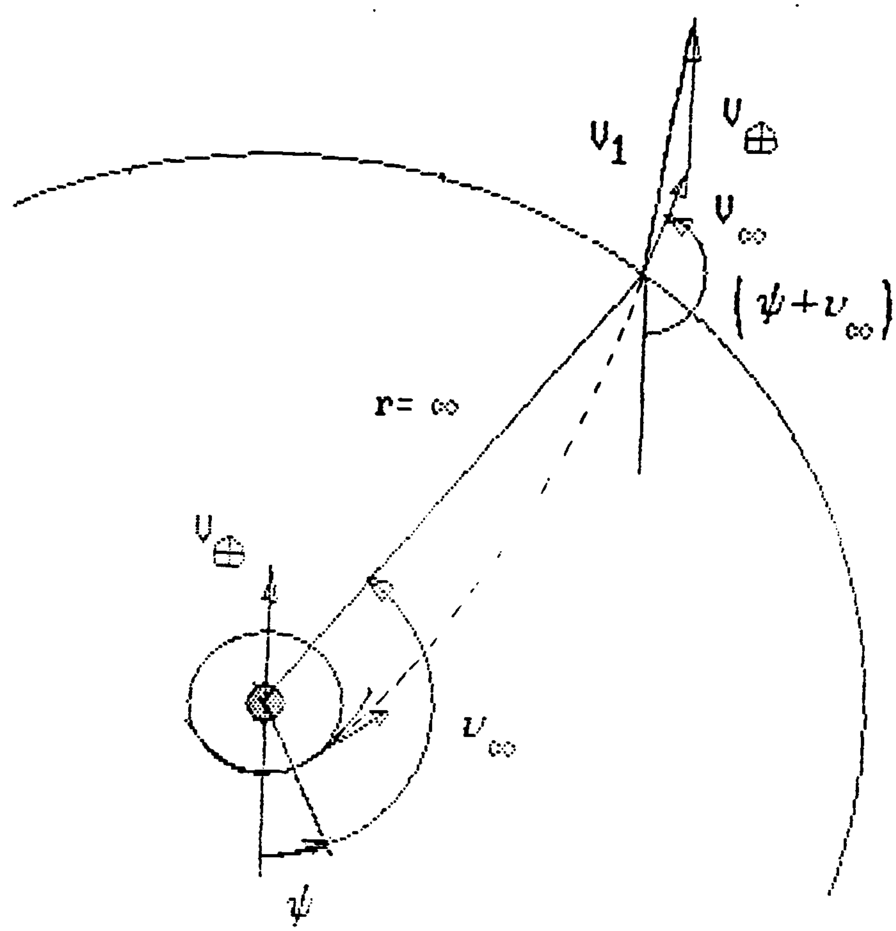


Figure 4.1 Relation between impulsive conditions and initial conditions

and the specific mechanical energy and angular momentum are, respectively:

$$\mathcal{E} = \frac{v_p^2}{2} - \frac{\mu}{r_p} \quad \text{and} \quad h = r_p \, v_p .$$

Where the subscript $_p$ denotes the parameters of the initial parking orbit. The angular momentum is simplified to this form under the assumption of a circular parking orbit and a tangential impulsive burn. Noting that the gravitational parameter of the Earth is μ_e, the total initial velocity of the spacecraft with a circular parking orbit and a tangentially applied escape impulse is

$$v_p = \Delta V_1 + \sqrt{\frac{\mu_e}{r_p}} .$$

Where r_p is the initial circular parking orbit radius.

Assuming no further thrusting after ΔV_1 until crossing the sphere of influence, the total energy is conserved as

$$\mathcal{E}_p = \mathcal{E}_\infty ,$$

so

$$\frac{v_p^2}{2} - \frac{\mu}{r_p} = \frac{v_\infty}{2} - \cancel{\frac{\mu}{r_\infty}}^{\,0}$$

or

$$v_\infty{}^2 = v_p{}^2 - 2 \frac{\mu_e}{r_p} .$$

The departure angle with respect to the local horizontal is

$$\text{angle}_{dep} = \psi + \nu_\infty ,$$

where

$$\nu_\infty = \cos^{-1}\left(- \left(1 + \frac{2\mathcal{E}h^2}{\mu^2} \right)^{-1/2} \right)$$

and ψ is an input variable. The initial r, v, and u conditions

are $r(0) = r_\oplus$, which is approximated as the Earth's radial distance from Sun; $u(0) = v_\infty \cos(\nu_\infty + \psi - \pi/2)$, which is the radial component; and $v(0) = v_\oplus + v_\infty \sin(\nu_\infty + \psi - \pi/2)$, which is the tangential component added to the Earth's orbital velocity. The initial Lagrange conditions are free to enable the optimization of the problem:

$$\lambda_r(0) = \text{free} \qquad \lambda_u(0) = \text{free} \qquad \lambda_v(0) = \text{free.}$$

FINAL CONDITIONS:

The final conditions are also related to an impulse (ΔV_2) but there is no equivalent ψ. The final conditions $r(t_f)$, $u(t_f)$, and $v(t_f)$ define the final required ΔV_p which will be adjusted by the optimization to match the given ΔV_2. The formulation from u and v to ΔV_p is as follows using Figure 4.2. At the boundary of Mars' SOI the orbital velocity of Mars must be subtracted to give the velocity with respect to Mars as

$$v_b = \sqrt{((v - \sqrt{\mu_m / r_f})^2 + u^2)} .$$

As with the Earth escape, it is assumed that no propulsive impulses will be used until the desired perimartem is reached r_p. It is also assumed that the flight may be phased so that this perimartem will be reached. The subscript p will denote the state at perimartem. Consequently, once again the energy of the orbit remains constant, $\mathcal{E}_p = \mathcal{E}_b$, so that the final velocity at perimartem is described by

$$v_p^2 = v_b^{\,2} + 2 \frac{\mu_{mars}}{r_p} .$$

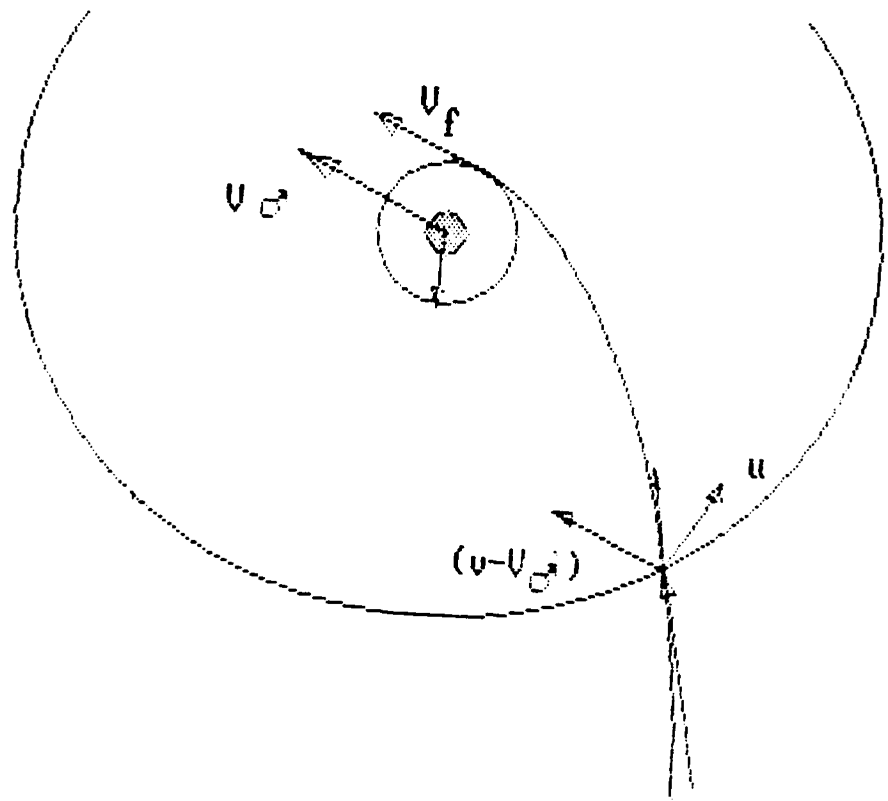


Figure 4.2 Relation of final conditions to impulsive conditions

And the ΔV_p required to brake into the circular parking orbit is

$$\Delta V_p = v_p - \sqrt{\frac{\mu_{mars}}{r_p}} \ .$$

Consequently, the final required condition, denoted by Ψ_1 is:

$$\Psi_1 = \sqrt{u^2 + (v - v_{mars})^2 + 2\,\frac{\mu_{mars}}{r_p}} - \sqrt{\frac{\mu_{mars}}{r_p}} - \Delta V_{tot} + \Delta V_1 = 0.$$

However, the condition may be rewritten in a more convenient form as

$$\Psi_1 = u^2 + (v - v_{mars})^2 + 2\,\frac{\mu_{mars}}{r_p} - \left(\sqrt{\frac{\mu_{mars}}{r_p}} + \Delta V_2\right)^2 = 0.$$

This form will lend itself to greatly simplifying the final Lagrange multiplier conditions as well as the numerical derivations. These final Lagrange multipliers are defined by

$$\lambda(t_f) = \left(\frac{\partial \varepsilon}{\partial x} + \nu^T \frac{\partial \Psi}{\partial x}\right)^T_{t=t_f} \ .$$

And, since the radius is to be maximized in the inner loop, $\varepsilon = r(t)$ is evident. Denoting that the differential velocity at the final solar radius is $v_{circ} = \sqrt{\mu / r(t_f)}$. Thus the final Lagrange conditions are

$$\lambda_r(t_f) = 1 + \nu_1 (v_f - v_{circ}) \sqrt{\mu} \,/\, r(t_f)^{3/2},$$

$$\lambda_u(t_f) = 2\,\nu_1\, u \ ,$$

$$\lambda_v(t_f) = 2\,\nu_1\, (v - v_{circ}).$$

These may be combined by noting that

$$\nu_1 = \frac{\lambda_u}{2u} = \frac{\lambda_v}{2(v_f - v_{circ})},$$

so that $$\lambda_r(t\ ) = 1 + \lambda_u(t_f)/2u\ \frac{(v-v_{mars})}{u}\ .$$

Hence a second final condition may be defined as

$$\Psi_2 = \lambda_r(t_f) - 1 - \lambda_v(t_f)\sqrt{\mu}/(2\ r_f^{3/2}),$$

and the third final condition as

$$\Psi_3 = \lambda_u(t_f)\ (v(t_f) - v_{circ}) - \lambda_v(t_f)\ u(t_f) = 0.$$

IN SUMMARY:

STATE VECTOR

DIFFERENTIAL EQUATIONS

$$X = \begin{bmatrix} r \\ u \\ v \\ \lambda_r \\ \lambda_u \\ \lambda_v \end{bmatrix} \qquad f = \dot{X}$$

EQUATIONS OF MOTION

$$\dot{r} = u$$

$$\dot{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + a(t)\ \frac{\lambda_u}{\sqrt{\lambda_u^{\ 2} + \lambda_v^{\ 2}}}.$$

$$\dot{v} = -\frac{uv}{r} + a(t)\ \frac{\lambda_v}{\sqrt{\lambda_u^{\ 2} + \lambda_v^{\ 2}}}$$

where $a(t) = \dfrac{T}{m_o - \mathring{m}\, t}$

LAGRANGIAN EQUATIONS

$$\dot{\lambda}_r = -\lambda_u \left(-\frac{v^2}{r^2} + \frac{2\mu}{r^3}\right) - \lambda_v \left(\frac{uv}{r^2}\right)$$

$$\dot{\lambda}_u = -\lambda_r + \lambda_v \left(\frac{v}{r}\right)$$

$$\dot{\lambda}_v = -\lambda_u \left(\frac{2v}{r}\right) + \lambda_v \left(\frac{u}{r}\right)$$

CONTROL LAW

$$\tan(\phi) = \frac{\lambda_u}{\lambda_v}$$

INITIAL CONDITIONS

$r(0) = r_{\oplus}$ (Earth's radial distance from Sun)

$u(0) = v_f \cos(\nu_f + \psi - \pi/2)$ (radial component)

$v(0) = v_{\oplus} + v_f \sin(\nu_f + \psi - \pi/2)$ (tangential component)

$\lambda_r(0) = \text{free}$ $\lambda_u(0) = \text{free}$ $\lambda_v(0) = \text{free}$

FINAL CONDITIONS

$$\Psi_1 = u^2 + (v-v_{mars})^2 + 2\frac{\mu_{mars}}{r_p} - \left(\sqrt{\frac{\mu_{mars}}{r_p}} + \Delta v_2\right)^2 = 0.$$

$$\Psi_2 = \lambda_r(t_f) - 1 - \lambda_v(t_f)\sqrt{\mu}/(2\, r_f^{3/2})$$

$$\Psi_3 = \lambda_u(t_f)\,(v(t_f) - v_{circ}) - \lambda_v(t_f)\,u(t_f) = 0.$$

<u>NUMERICAL SOLUTION:</u>

The equations of motion and optimization derived above will be solved numerically by the method presented in the previous chapter. As in that case, the solar orbit radius will be maximized for a certain time period, t_o to t_f, and an outer loop will adjust the final radius to that of Mars thus giving the minimum transfer time. Since the differential equations are the same, the equations of variation are also identical to those of chapter 3. The A matrix is also the same $\mathbf{A}(t) = \partial f/\partial \mathbf{x}$. As is,

$$\dot{\Phi}(t,t_0) = \mathbf{A}(t)\,\Phi(t,t_0),$$

with initial conditions $\Phi(t_0,t_0) = I$. (identity matrix.) Remembering that the Φ matrix (called the *state transition matrix*) will propagate the initial deviation from the $\mathbf{x}_o$ trajectory to any time in the integration,

$$\delta\mathbf{x}(t) = \Phi(t,t_0)\,\delta\mathbf{x}(t_0).$$

Again this system is a two point boundary problem and again

the 'shooting method' will be used. The corrected initial condition is found by the same method as that shown in Chapter 3:

$$\delta x(t_1) = [(B\ \Phi(t_2,t_1)]^{-1}\ G.$$

For this problem the error vector is $G = [\Psi]^T$

where $\Psi_1 = u^2 + (v-v_{mars})^2 + 2\,\dfrac{\mu_{mars}}{r_p} - \left(\sqrt{\dfrac{\mu_{mars}}{r_p}} + \Delta V_2\right)^2 = 0,$

$$\Psi_2 = \lambda_r(t_f) - 1 - \lambda_v(t_f)\ \sqrt{\mu}/(2\ r_f^{\ 3/2})\ ,$$

$$\Psi_3 = \lambda_u(t_f)\ (v(t_f) - v_{circ}) - \lambda_v(t_f)\ u(t_f) = 0.$$

Therefore, the $B = \delta G/\delta x$ matrix is

$$\begin{bmatrix} 2V_dV_c & 2u & 2\ V_d & 0.0 & 0.0 & 0.0 \\ \dfrac{3\lambda_v\sqrt{\mu}}{4\ r^{2.5}} & 0.0 & 0.0 & 1.0 & 0.0 & -V_c \\ \lambda_u V_c & -\lambda_v & \lambda_u & 0.0 & V_d & -u \end{bmatrix}$$

where $V_d = v(t_f) - \sqrt{\mu/r(t_f)}$ and $V_c = 1/2\ \sqrt{\mu}/r(t_f)^{1.5}$.

Again, once B and $\Phi(t_2,t_1)$ are determined, multiplied, and inverted with the error vector, the improved corrections, $\delta\lambda^T$, are added to the variable initial conditions (in this case the three initial Lagrange multipliers).

$$\lambda^T_{new} = \lambda^T_{initial} + \delta\lambda^T(t_1)$$

This procedure is again iterated until the error vector reaches an acceptable minimal amount or diverges to infinity. The latter case indicates that a better initial guess that is 'close' to the actual optimal initial conditions is required.

ADJUSTING FOR MINIMUM TIME OF FLIGHT:

The above procedure will maximize the final orbital radius for the given time of flight t_f. To adjust this to find the minimum time to reach a certain orbit radius, in this case Mars' solar orbit, a simple outer loop may be written, as in the previous section, using a scalar Newton-Raphson procedure to adjust the time of flight as

$$t_{f_{k+1}} = t_{f_k} + \frac{(t_{f_k} - t_{f_{k-1}})}{r(t_{f_k}) - r(t_{f_{k-1}})}\,[r_f - r(t_{f_k})],$$

where $r_f = r_{Mars}$.

NORMALIZATION OF SPACECRAFT PARAMETERS:

To allow for ease of programming and more general results, the physical parameters of the spacecraft will be normalized. Defining mass ratios as the subsystem weight divided by the the initial total spacecraft mass in low earth orbit (LEO):

$\mu_L = M_L$ (payload mass)/ M_{Leo},

$\mu_W = M_W$ (propulsion systems)/ M_{Leo} ,

$\mu_{IP} = M_{IP}$ (ion propellant) / M_{Leo},

$\mu_{P1} = M_{P1}$ (propellant for Δv_1) / M_{Leo},

and $\mu_{P2} = M_{P2}$ (propellant for Δv_2) / M_{Leo}.

These subsystems are shown in Figure 4.3. The propulsion systems may be further divided into the *high thrust system* and the low thrust or *power system*:

$$\mu_W = \mu_{WH} + \mu_{WL}$$

where μ_{WH} is the reactor and controls, turbopump, piping, nozzle, thrust structure, etc.; and μ_{WL} is the power producing equipment *excluding reactor*: turbine, radiator, working fluid, piping, and electric engines. To adjust for the fact that the power producing system does not include the reactor (since the high thrust required will normally define the reactor size) we define μ_R as the mass ratio of reactor where $\mu_R = \rho\ \mu_{WH}$ and ρ is the reactor mass factor.The total mass of the required *power* producing system (including reactor) is μ_T; where the required portion of the required reactor is

$\mu_{Rpwr} = \gamma\ \mu_T$ and γ = reactor specific mass.

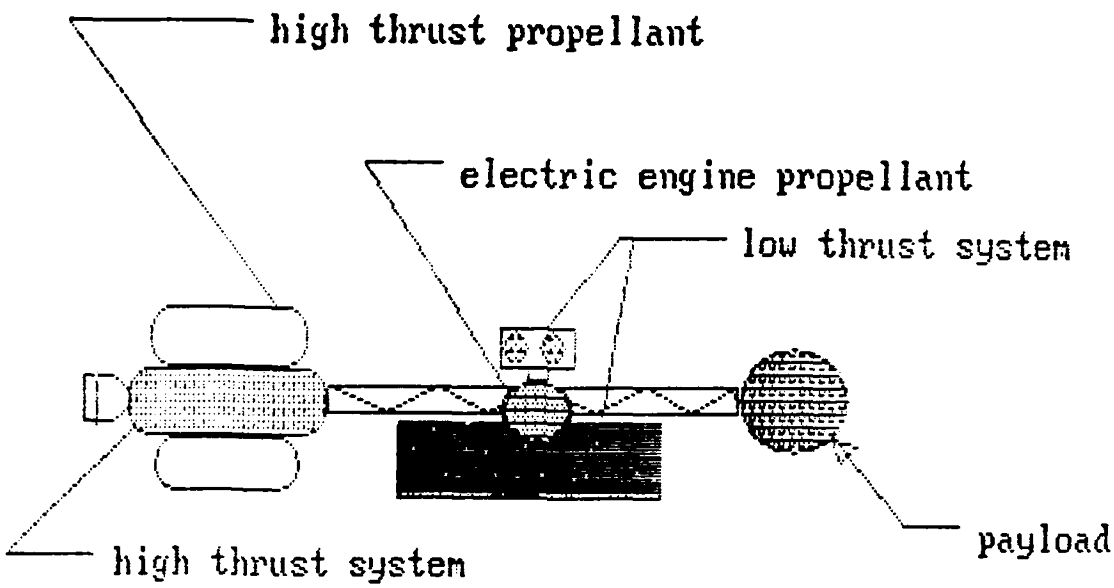


Figure 4.3 Spacecraft subsystems

Now assuming that the high thrust reactor may be used in a closed loop mode and, therefore, saves the necessity of another reactor for the power generation mode the actual power unit mass ratio is

$$\mu_{WL} = \mu_T - \mu_{Rpwr} ,$$

or

$$\mu_{WL} = \mu_T(1 - \gamma),$$

where $\mu_{Rpwr} \leq \mu_R$.

The following summarizes the mass ratios and their related parameters.

SUMMARY OF MASS RATIOS AND PARAMETERS:

(Note: all masses compared to the spacecraft's mass in low Earth orbit M_{Leo})

μ_L : mass ratio of the payload

μ_W : mass ratio of both propulsion systems

μ_{IP}: mass ratio of the electric propellant

μ_{P1}: mass ratio of the propellant for Δv_1

μ_{P2}: mass ratio of the propellant for Δv_2

μ_{WH}: mass ratio of the reactor and equipment for high thrust operation

μ_{WL}: mass ratio of the low thrust system equipment *excluding reactor*

μ_R :mass ratio of reactor only

ρ :the reactor mass factor (portion of high thrust system that is reactor)

μ_T : mass ratio of the required low thrust producing system (including reactor)

γ : reactor specific mass ratio (portion of low thrust system that is reactor)

μ_{Rpwr}: mass ratio of reactor required for low thrust

RESULTING RELATIONS:

$$\mu_W = \mu_{WH} + \mu_{WL} \qquad \mu_R = \rho\, \mu_{WH}$$

$$\mu_{Rpwr} = \gamma\, \mu_T \qquad \mu_{WL} = \mu_T - \mu_{Rpwr} = \mu_T(1 - \gamma)$$

$$\mu_{Rpwr} \le \mu_R$$

How efficiently the high thrust portions of the propulsion unit perform may be measured by the high thrust/wt ratio β: which is the amount of high thrust possible per unit weight of the propulsion system as described by

$$\beta = T/M_{WH}g_o = \frac{\mathring{m}_H V_H}{M_{WH} g_o} = \frac{\mu_{HDOT} V_H}{\mu_{WH}\, g_o}\ .$$

The low thrust system's performance is measured by the specific power ratio of power system α which is the reactor output power (kWe) / [power producing mass (kg)] or $\alpha = W_e / M_T$ where M_T is the required power producing mass.

The exhaust velocities and mass flow rates of the high and low thrust modes are also defined as V_L, the exhaust velocity of the low thrust engines(km/sec); $\mathring{m}_L$,the mass flow rate of the low thrust engines (kg/sec), V_H, the exhaust velocity of the high thrust engines(km/sec); and $\mathring{m}_H$, the mass flow rate of the high thrust engines (kg/sec). The mass flow rates may also be

normalized as

$$\mu_{LDOT} = \mathring{m}_L \, / \, M_{Leo} \qquad \text{and} \qquad \mu_{HDOT} = \mathring{m}_H \, / \, M_{Leo}.$$

Finally, the time of flight will be noted by τ.

We will next normalize the low thrust acceleration term to be used in the optimization:

$$a(t) = \frac{T}{m_o - \mathring{m}_L t} \, .$$

By noting that $T = \mathring{m}_L V_L$ and dividing thru by $m_o = M_{esc}$ (mass of space craft at escape from Earth)the term becomes

$$a(t) = \frac{(\mathring{m}_L/m_{esc}) V_L}{1 - (\mathring{m}_L/m_{esc}) t} \, .$$

Consequently, a relation between $\mathring{m}_L/m_{esc}$ and the normalized spacecraft parameters defined previously is needed. The beam power of the ion engines W_{beam} is defined by

$$W_{beam} = W_{generated} = \eta \, \alpha \, M_T = 1/2 \left[\frac{M_{IP}}{\tau} \right] V_L^2 \quad (*).$$

In this case, the efficiency η will be assumed to be 100 %.
The beam power relates to the thrust by

$$T = 2 \, W_{beam} / \, V_L = \mathring{m}_L \, V_L,$$

thus

$$\mathring{m}_L = \frac{2\ \alpha\ M_T}{V_L^2} \ .$$

Dividing thru by M_{esc} and realizing $M_{esc} = M_{Leo} - M_{P1,}$ and then using M_{Leo} on the right hand side, this may be transformed into a function of mass ratios as

$$\frac{\mathring{m}_L}{M_{esc}} = \frac{2\alpha}{V_L^2}\frac{M_T}{M_{esc}} = \frac{2\alpha}{V_L^2}\frac{\mu_T}{1 - \mu_{P1}} \ .$$

Additionally, the mass ratio of the required low thrust propellant is defined from (*) as

$$\mu_{IP} = \frac{2\alpha}{V_L^2}\,\tau\ \mu_T.$$

According to Stuhlinger [8: p. 79], a good guess for the optimal low thrust exhaust velocity V_L is $V_{L_{opt}} = \sqrt{\alpha\ \tau}$. If this assumption is used the mass ratio becomes

$$\mu_{IP} = 2\ \mu_{T.}$$

Now the impulsive terms will be normalized. Using the Tsiolkovskii equation,

$$e^{\Delta v\ /\ V_H} = M_{initial} /\ M_{final} \ ,$$

the high thrust burn to escape LEO (Δv_1) and the high thrust braking at Mars parking orbit (Δv_2) can be stated as

$$\Delta v_1 = V_H \ln [M_{Leo} / M_{esc}] ,$$

and
$$\Delta v_2 = V_H \ln \left[\frac{M_L + M_W + M_{P2}}{M_L + M_W} \right],$$

respectively. Dividing thru the masses by M_{Leo} we get

$$\Delta v_1 = V_H \ln \left[\frac{1}{1 - \mu_{P1}} \right],$$

and
$$\Delta v_2 = V_H \ln \left[\frac{1 - \mu_{IP} - \mu_{P1}}{\mu_L + \mu_W} \right] ,$$

by noting that $\mu_L + \mu_W + \mu_{IP} + \mu_{P1} + \mu_{P2} = 1.$

In summary, the following equations and input variables are required for the normalized equations:

$$a(t) = \frac{(\dot{m}_L / m_{esc}) V_L}{1 - (\dot{m}_L / m_{esc}) t}$$

where
$$\frac{\dot{m}_L}{M_{esc}} = \frac{2\alpha}{V_L^2} \frac{\mu_T}{1 - \mu_{P1}} , \text{ and } \mu_{IP} = \frac{2\alpha}{V_L^2} \tau \mu_T .$$

If the optimal exhaust velocity is assumed as $V_{L_{opt}} = \sqrt{\alpha \tau}$, then $\mu_{IP} = 2 \mu_T$. The high thrust parameters are normalized as

$$\Delta v_1 = V_H \ln \left[\frac{1}{1 - \mu_{P1}} \right],$$

and
$$\Delta v_2 = V_H \ln \left[\frac{1 - \mu_{IP} - \mu_{P1}}{\mu_L + \mu_W} \right]$$

remembering that $\mu_W = \mu_{WH} + \mu_T(1-\gamma)$. Consequently, to perform the optimization the following constants and mass rations must be assumed: [μ_L, μ_{P1}, β, α, V_L , V_H, ρ, γ, μ_T, τ]. However, if $v_{L_{opt}} = \sqrt{\alpha \tau}$ is assumed τ will not be needed.

LIMITS ON [μ_L, μ_{P1}, V_L , V_H, α, β, ρ, γ, μ_T]:

Assumed mass ratios μ_{P1}, μ_L and μ_W:

Due to the requirement that the spacecraft be capable of escaping Earth orbit and braking into Mars orbit, there is a minimum and maximum amount of Δv_1 propellant mass that may be used. The minimum Δv_1 and Δv_2 (for Earth escape and Mars capture, respectively) may be determined simply by

$$\Delta v_{esc} = v_{esc} - v_{circ}$$

or $$\Delta v_{esc} = \sqrt{2\mu/r} - \sqrt{\mu/r} = (\sqrt{2} - 1)\sqrt{\mu/r} .$$

The same is true for a braking Δv_{brake}. The minimum μ_{P1} is determined easily using the Tsiolkovskii equation:

$$\mu_{P1min} = \left[1 - e^{(-\Delta v_{esc}/ v_H)} \right] ,$$

assuming V_H is known. Using the minimum for M_{P2},

$$M_{P2min} = [M_L + M_W] \left[e^{(\Delta v_{brake}/ V_H)} - 1 \right] ,$$

the maximum μ_{p1} is found:

$$\mu_{P1max} = 1 - \mu_{IP} - [\ \mu_L + \mu_W\ ] \left[\ e^{\left[\Delta v_{brake} / v_H\ \right]}\ \right] .$$

Propulsion unit low/high exhaust velocities V_L and V_H:

The exhaust velocities of the low and high thrusting modes are technology dependent. However, some ranges of values are:

	System Type		Exhaust Velocity (km/sec)
V_L:			
	Ion and MFD systems	:	30 to 250
	Electrothermal	:	4 to 20
V_H:	Nuclear	:	8 to 10 .

For actual explanations of these systems see Appendix A.

The specific power α:

The specific power of a *bimodal* propulsion unit is also technology dependent. At present a seemingly possible range is roughly from 0.002 to 0.005 kWe/kg. In this case the the mass includes the reactor, nozzle, thrust structure, propellant feeds, turbomachinery, radiators, and ion engines. Again refer to Appendix A for more information.

Thrust-to-weight ratio of the propulsion unit β :

The thrust of the propulsion system must be sufficient to enable the spacecraft to escape Earth orbit. Additionally, the assumption that this nuclear thrusting is effectively impulsive (instantaneous) should be checked. This may be easily accomplished using the equations of motion defined in Chapter 3 and adjusting them to provide the thrusting always tangential to the initial circular parking orbit (i.e. circumferential thrust) in an Earth centered two body problem. These equations may now be integrated using Haming's Method also described in Section 3. The impulsive case of the high thrust system may be found by assuming an instantaneous Δv for the initial conditions and equations of motion. The actual performance may be found in a similar way by assuming the appropriate a(t) and integrating the equations of motion.

By comparing

$$\frac{\text{time of flight (actual)}}{\text{time of flight (impulsive)}},$$

(both to the earth's sphere of influence) to the ratio

$$\frac{\beta\ \mu_{WH} g_o}{V_H} = \mu_{Hdot}$$

which is the specific mass flow rate, we see how close the actual thrusting compares with the impulsive thrusting. (see figures 4.4 and 4.5) The first figure clearly shows that above a value for $\beta\mu_{WH}$ of .2 the range of exhaust velocities for nuclear rocket engines has little effect. The next figure shows the effect of

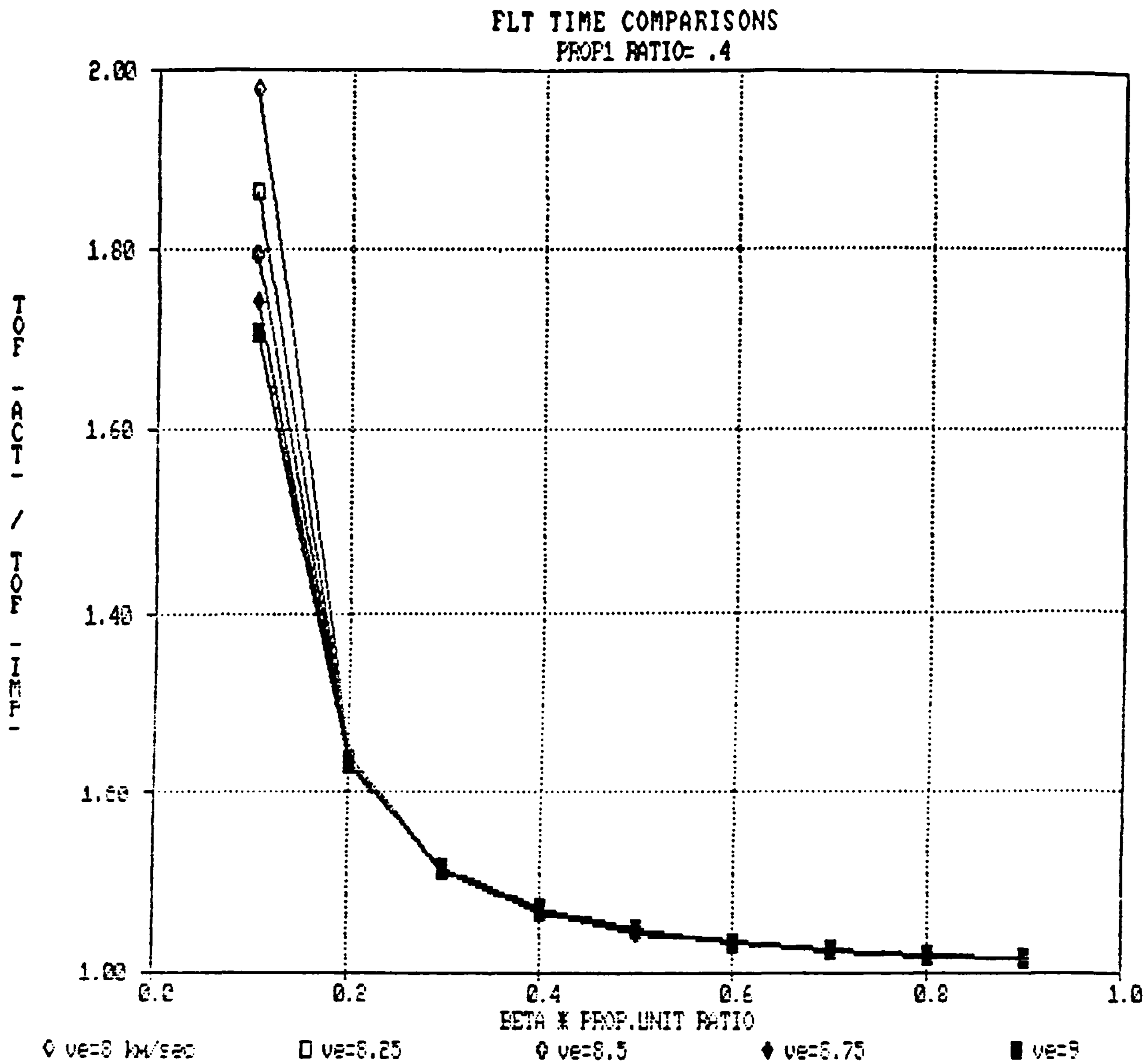


Figure 4.4 Effect of exhaust velocity on time-of-flight ratio verses $\beta * \mu_{WH}$

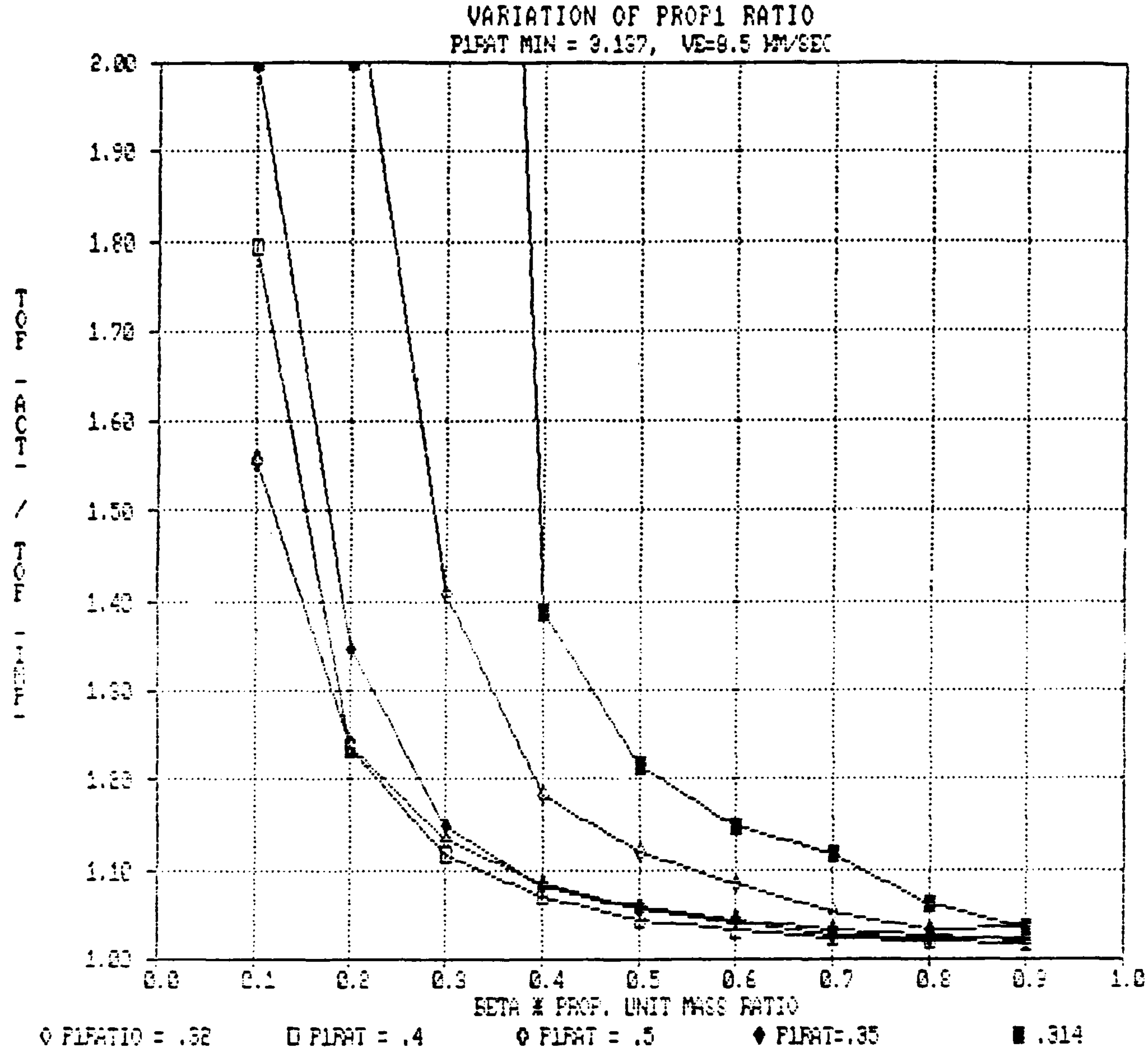


Figure 4.5 Effect of excess escape propellant (μ_{P1}) on time-of-flight ratio verses $\beta * \mu_{WE}$

excess fuel on the time of flight. It is evident that unless we are very close to the minimum μ_{pi} the curves approach a limit. Assuming that such excess fuel is available and allowing for an approximate time of flight error of around five percent we shall require that

$$\beta\ \mu_{WH} \geq .5 \quad \text{or} \quad \mu_{WH} \geq 1/\ (2\ \beta),$$

since an increased propulsion unit ratio will be required depending upon the technical capabilities of the system as described by β. Without this requirement, there are some low $\beta\mu_{WH}$ values that are not sufficient to even allow the spacecraft to escape orbit much less permit the assumption of impulsive thrusting. This can be explained by the velocity and gravity losses experienced by a spacecraft during a thrusting of finite time.

Reactor mass and specific mass factors ρ and γ:

These factors are also technology dependent. Depending upon the nuclear rocket system, the portion that is reactor is from seventy to eighty percent, ρ: 0.70 to 0.80. On the other hand, the percentage of the total power producing mass that is reactor is only from 20 to 30 percent, γ: .20 to .30. For a *non*-bimodal system, i.e. separate reactors for the high thrust and power generation systems, one may set

$$\gamma = 0.0 \qquad \text{or} \qquad \mu_{WL} = \mu_T.$$

Power system mass ratio μ_T:

The power system mass ratio defines how much power is generated and, in turn, the low thrust available to the spacecraft. Since it is assumed that reactor size will be defined by the high thrust mode, the upper limit on μ_T is

$$\mu_T \le \frac{\mu_{Rpwr}}{\gamma} \le \frac{\mu_R}{\gamma} = \frac{\rho}{\gamma}\, \mu_{WH} = \frac{\rho}{\gamma}\, \frac{V_H}{2\,\beta}$$

since $\mu_{Rpwr} = \gamma\, \mu_T$. The lower limit occurs when there is no ion drive system present: $\mu_T \ge 0.0$.

PARAMETERS TO VARY FOR OPTIMIZATION:

Of the multiple parameters given [μ_L, μ_{P1}, V_L , V_H, α, β, ρ, γ, μ_T], only μ_{P1} and μ_T can be freely varied without technological improvements or change of mission. In simple terms, the variation of μ_{P1} between the limits prescribed above will determine the optimum amount of excess high thrust fuel (and, consequently, ΔV) used either for escape from Earth or braking at Mars. The variation of μ_T on the other hand, will indicate the optimum proportion of low thrust drive and excess high thrust fuel. In summary,

variation of μ_{P1}:

$$\mu_{P1min} \le \mu_{P1} \le \mu_{P1max}$$

where $$\mu_{P1min} = \left[1 - e^{(-\Delta v_{esc} / V_H)} \right]$$

and $$\mu_{P1max} = 1 - \mu_{IP} - [\mu_L + \mu_W] \left[e^{\left[\Delta v_{brake} / V_H\right]} \right] .$$

variation of μ_T:

$$0 \le \mu_T \le \frac{\rho}{\gamma} \mu_{WH} = \frac{\rho}{\gamma} \frac{V_H}{2 \beta} .$$

These two variations will be fully explored in the next section.

CHAPTER V: OPTIMIZATIONS AND RESULTS

INTRODUCTION:

A minimum time, direct trajectory will be found for a spacecraft utilizing a bimodal nuclear power propulsion system as defined previously. Several different optimizations will be performed. First, the optimal vectoring $\phi(t)$ of the low-thrust electric engines will be determined. Simultaneously, the excess fuel for the nuclear engines will be divided in various portions (μ_{p1} and μ_{p2}) to vary the excess escape and arrival velocities. These two optimizations, including varying the injection angle ψ, will show how best to utilize an existing normalized spacecraft design by indicating the parameters for fastest time-of-flight.

After this trajectory/fuel division optimization is complete, the size of the electric power generator/electric engines/electric engine propellant, μ_T, will be varied. The variation of μ_T will only affect the amount of excess high thrust fuel. In other words, it will be shown to what extent the excess mass should be used for either the low thrust system or the high thrust fuel. Consequently, the advantage of the bimodal concept will be tested and the sizing of the electric drive relative to the rest of the spacecraft may be determined.

ASSUMED ORBITAL PARAMETERS:

As stated in the last chapter, the Earth and Mars will be assumed to be in coplanar, circular orbits. Additionally, it is

assumed that Mars will be in the correct position for the arriving spacecraft. For this optimization, the assumed parking orbits are 500 km and 1000 km for the Earth and Mars, respectively. It will also be assumed that these parking orbits are at the proper inclination so as not to require the spacecraft to make a plane change to attain the correct plane of the Earth-Mars transit.

SAMPLE VEHICLE PARAMETERS:

For the actual numerical optimization some sample vehicular parmeters must be assumed. For a space vehicle responsible only for carrying mission personnel (i.e.cargo for the Mars exploration will be carried by electrically propelled vehicles) as quickly as possible, a payload ratio of 10 percent seems reasonable. For the nuclear and electric propulsion systems existing technologies will be utilized. The thrust-to-weight ratio of the NERVA I nuclear rocket engine is 2.84 while the exhaust velocity is 8.25 Km/sec. (see figure 5.1).

Since no high power bimodal system has been designed, it will be assumed in this initial optimization that a separate power source shall be carried by the spacecraft to provide the electric engines with power, i.e. $\mu_{WL} = \mu_T$. In this case the SP-100 reactor will be used. For the low thrust system's combined reactor and electric engine mass, a specific power ratio of 0.05 kWe/kg seems reasonable for existing technologies. (Note: this specific power ratio is different than that defined in the last chapter since, in this case, it does not include the mass of the high thrust rocket/reactor unit.) The actual electric engines to

Stage weights (NERVA-I engines), kg (lb):	
Nuclear engine (includes internal shield and thrust structure)	11 680 (25 750)
External shielding (top nuclear stage only)	908 (2000)
Tank mass (dry), kg/module (lbm/module)	7070 (15 586) [+0.064 × M_{prop}]
Residuals (gas and liquid), kg/engine (lb/engine)	908 (2000)
Interstage structure ($M_{pay\ total}$)	0.015
Nuclear engine performance:	
Thrust, N (lb)	334 000 (75 000)
Specific impulse, sec	825

Figure 5.1 Nerva-I characteristics [13]

be used, whether based upon electrothermal, electrostatic, or electromagnetic acceleration; will greatly depend upon the optimal exhaust velocity determined by the optimization and the availability of such electric engines. However, due to the assumed specific power and the probable time of flight (approximately 200 days), the required exhaust velocities will fall around 40 km/sec, thus an ion electric engine system will probably be required. The summarized assumed parameters for the problem at hand are:

MASS RATIOS:

PAYLOAD	0.100
NUCLEAR PROPULSION UNIT	0.176
POWER GENERATION UNIT	0.050
ELECTRIC PROPELLANT	0.100
HIGH THRUST FUEL (REQ'D)	0.377
HIGH THRUST FUEL (EXCESS)	0.197.

The sizing of the low thrust propulsion unit, defined by μ_T, will be varied after the initial optimization of injection angle and fuel division. As a starting point for this first optimization a μ_T of 0.05 will be chosen. This results in a low thrust fuel ratio of 0.1 (since approximately optimal electric exhaust velocity is assumed by the numerical optimization), and an excess high thrust fuel ratio of 0.197 (meaning $\mu_{P1} + \mu_{P2} = \mu_{P1MIN} + \mu_{P2MIN} + 0.197$), which will allow for a substantial variation in the excess escape and braking velocities.

OPTIMAL INJECTION ANGLE AND HIGH THRUST FUEL DIVISION:

The first optimization to be attempted will be the variation of the Earth injection angle (ψ) and the variation of the high thrust fuel (μ_{p1}) between the Earth escape burn and the Mars

braking burn. For each specific case the program will determine the optimum thrust vectoring history to obtain the minimum flight time. The exact optimal combination of injection angle and fuel division will not be determined. Rather, a general search and three dimensional graph will be created to show the effects of the variations.

The three dimensional plot shown in Figure 5.2 shows the effect of variation of ψ and μ_{p1}. A well behaved sloping occurs with a minimal near $\psi = 0^{\circ}$ and approximately half the excess fuel going to each burn. The regions near the edges, especially near the 90% / +30 part of the graph should be ignored since no data was created for that region due to the non-convergence of the program. The three dimensional graphing routine incorrectly assumes a contour there. The topographical plot shown in Figure 5.3 allows for a better look at the optimal region which occurs at approximately$\psi = -5^{\circ}$ and 50 % fuel usage. The minimum flight time is approximately 110 days.

It is interesting to analyze the optimal low thrust vectoring at different regions of the injection angle / fuel division variation. At the minimum time of flight, the vectoring $\Phi(t)$ resembles very closely the one found previously for fastest transfer between circular orbits in chapter 3. (see figure 5.4) This equates roughly to accelerating the vehicle half the mission and decelerating the last half.

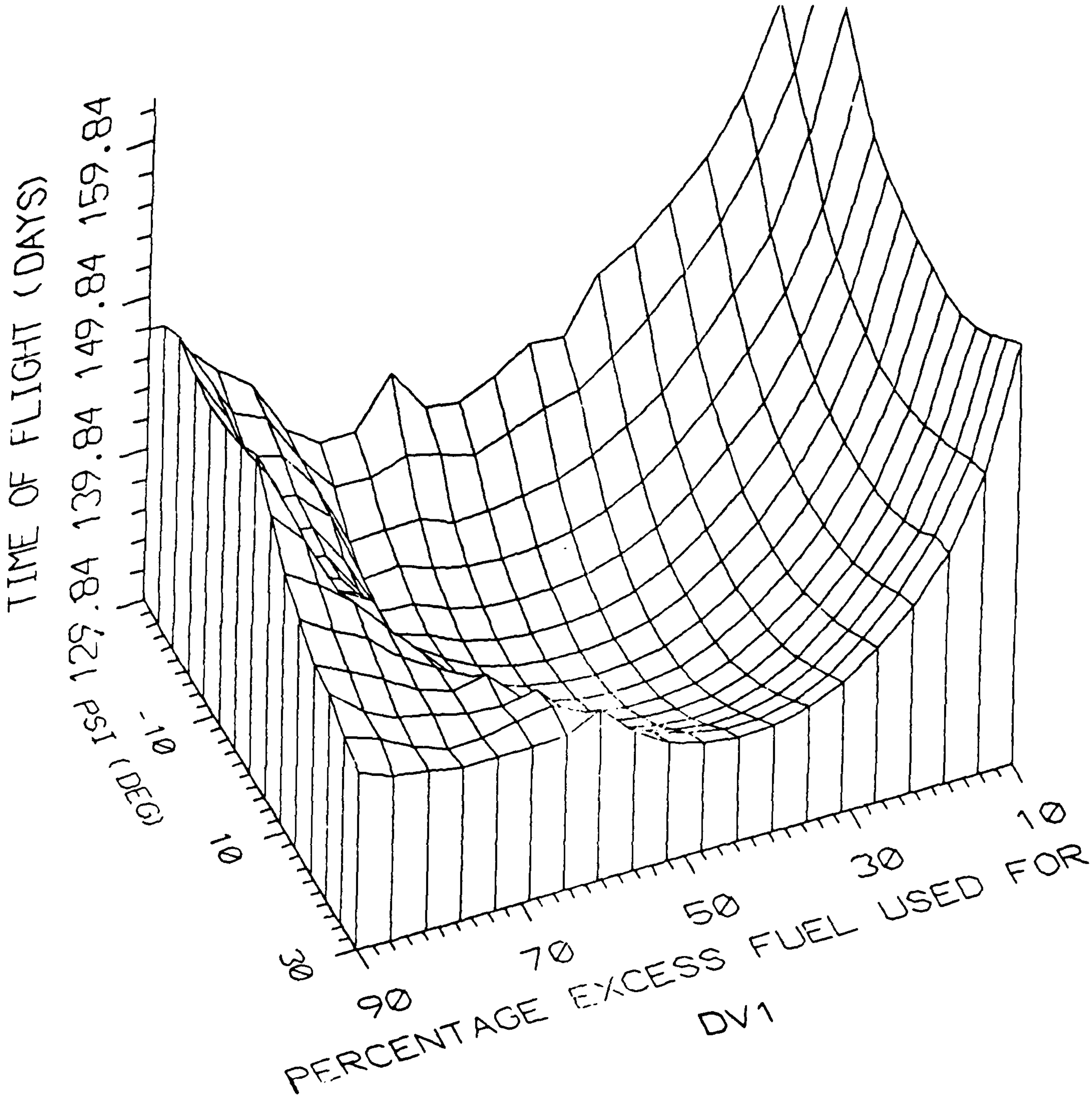


Figure 5.2 Three dimensional plot showing the effect of varying injection angle and excess fuel division on the time-of-flight

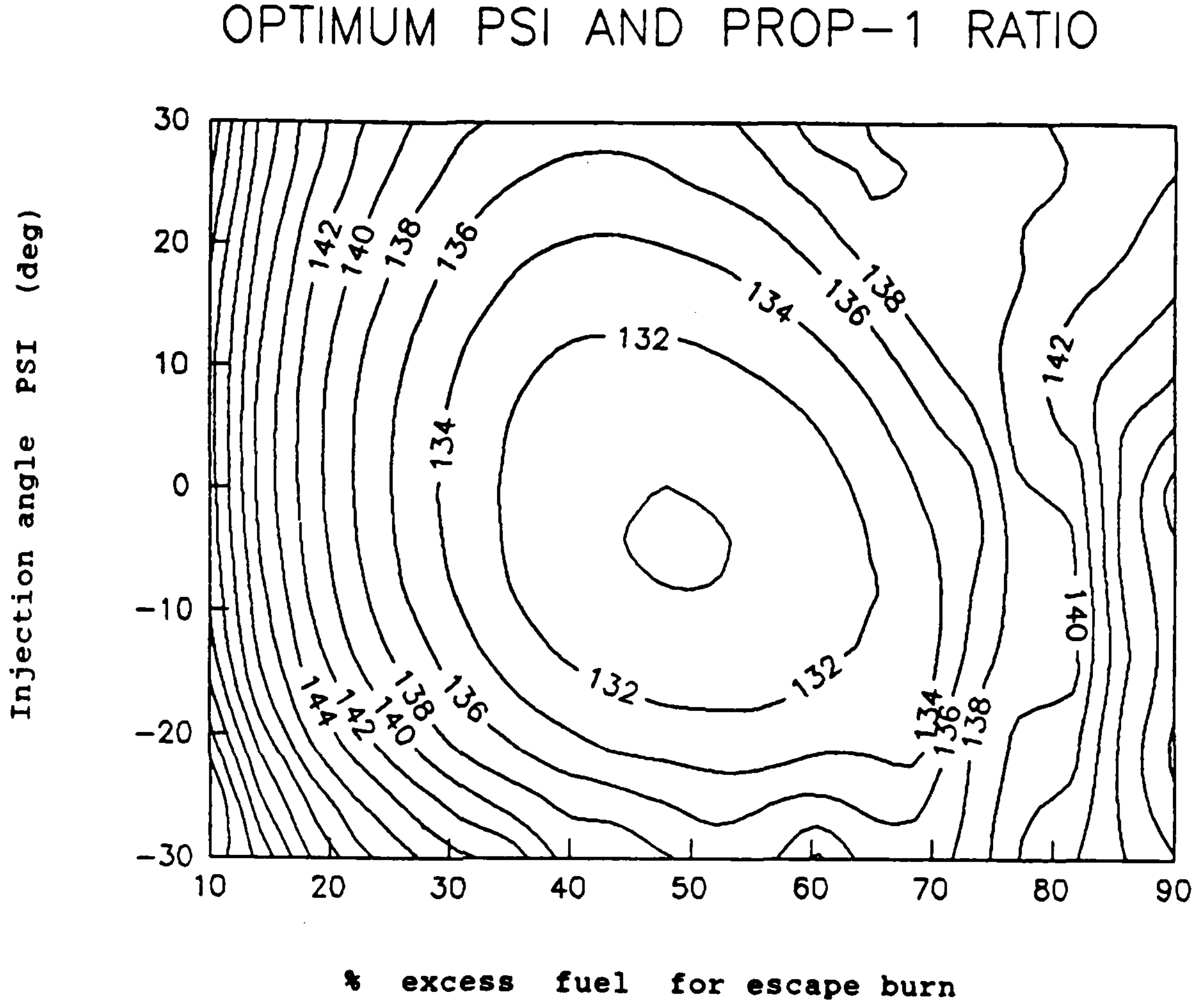


Figure 5.3 Topographical plot showing the effect of varying injection angle and excess fuel division on the time-of-flight

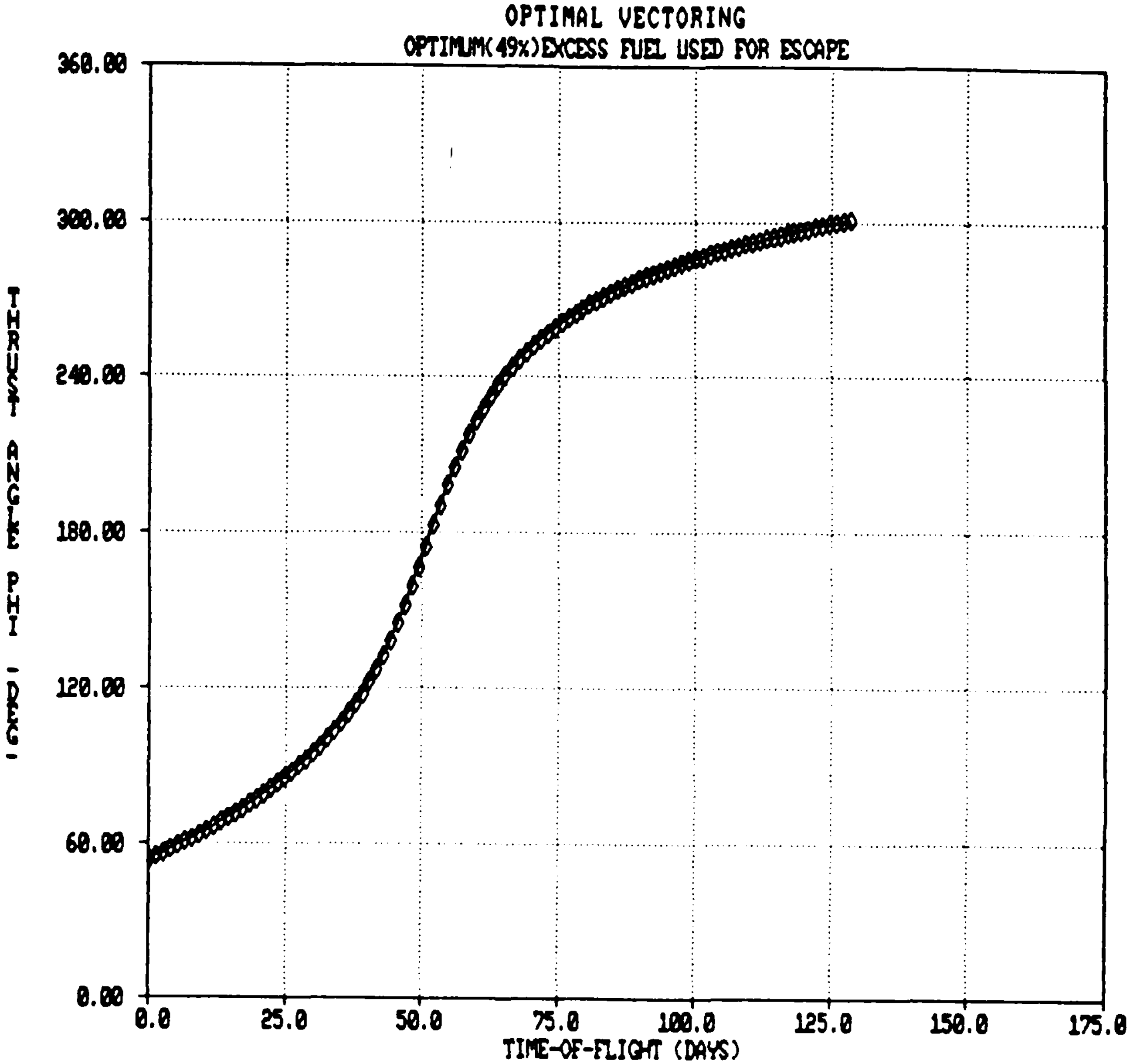
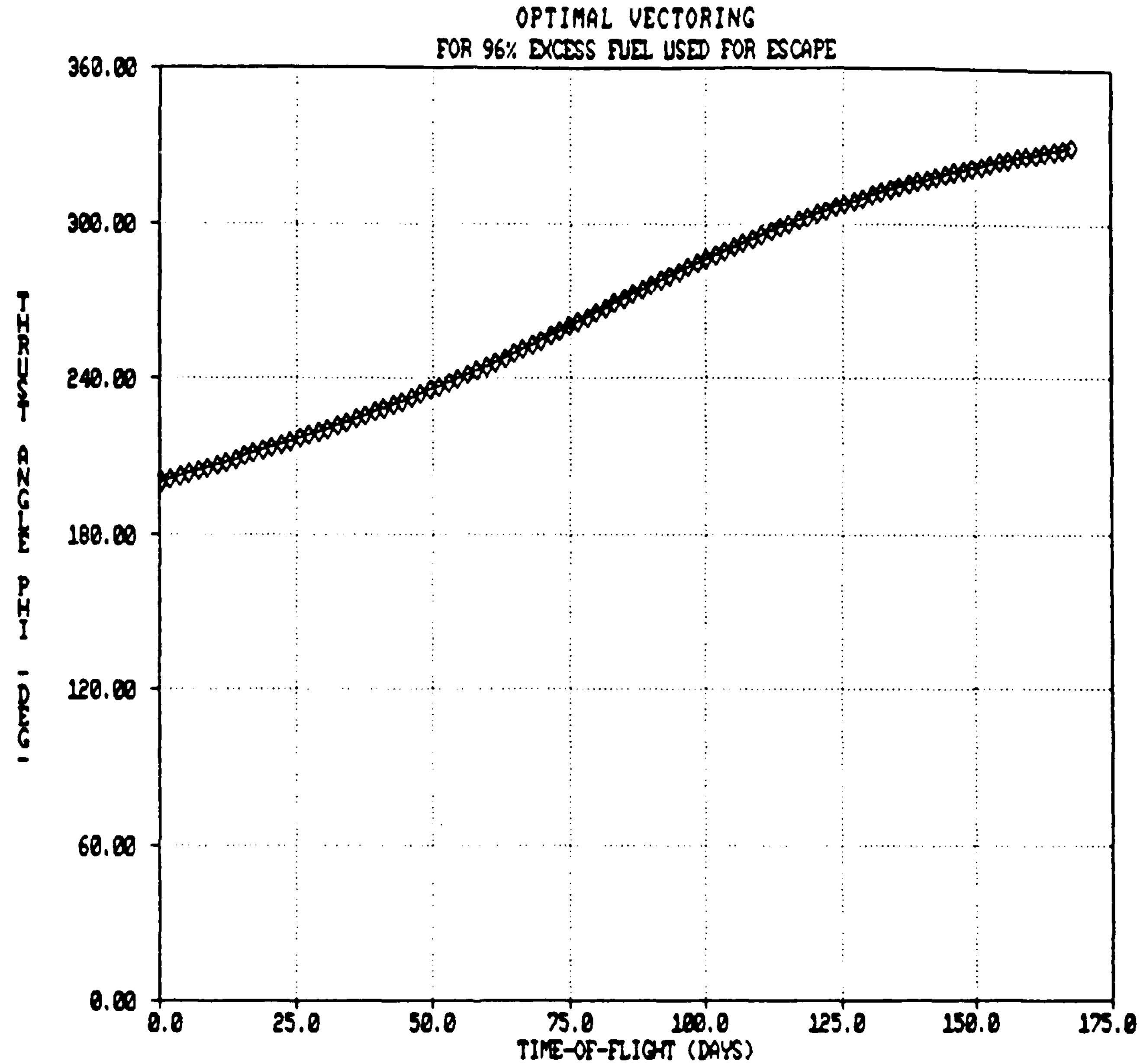


Figure 5.4 Optimal thrust vectoring for optimal fuel division

For the extremes of the fuel division the optimal vectoring may show why they do not provide the minimum time of flight. For the case where more excess fuel is used for the hyperbolic escape from Earth, it is clear from Figures 5.5 and 5.6 that the electric thrust system spends nearly all of the transit time decelerating the vehicle to a speed where the high thrust engines with the remaining fuel may give the ship sufficient braking ΔV. As may be expected, Figures 5.7 and 5.8 shows that if most of the fuel is saved for the braking into Mars orbit, the low thrust propulsion unit must accelerate most of the distance just to arrive at Mars. By using the optimal fuel division, up to thirty days of flight time may be saved even though no change in either propulsion unit is made.



5.5 Optimal thrust vectoring for 96% excess fuel used for escape

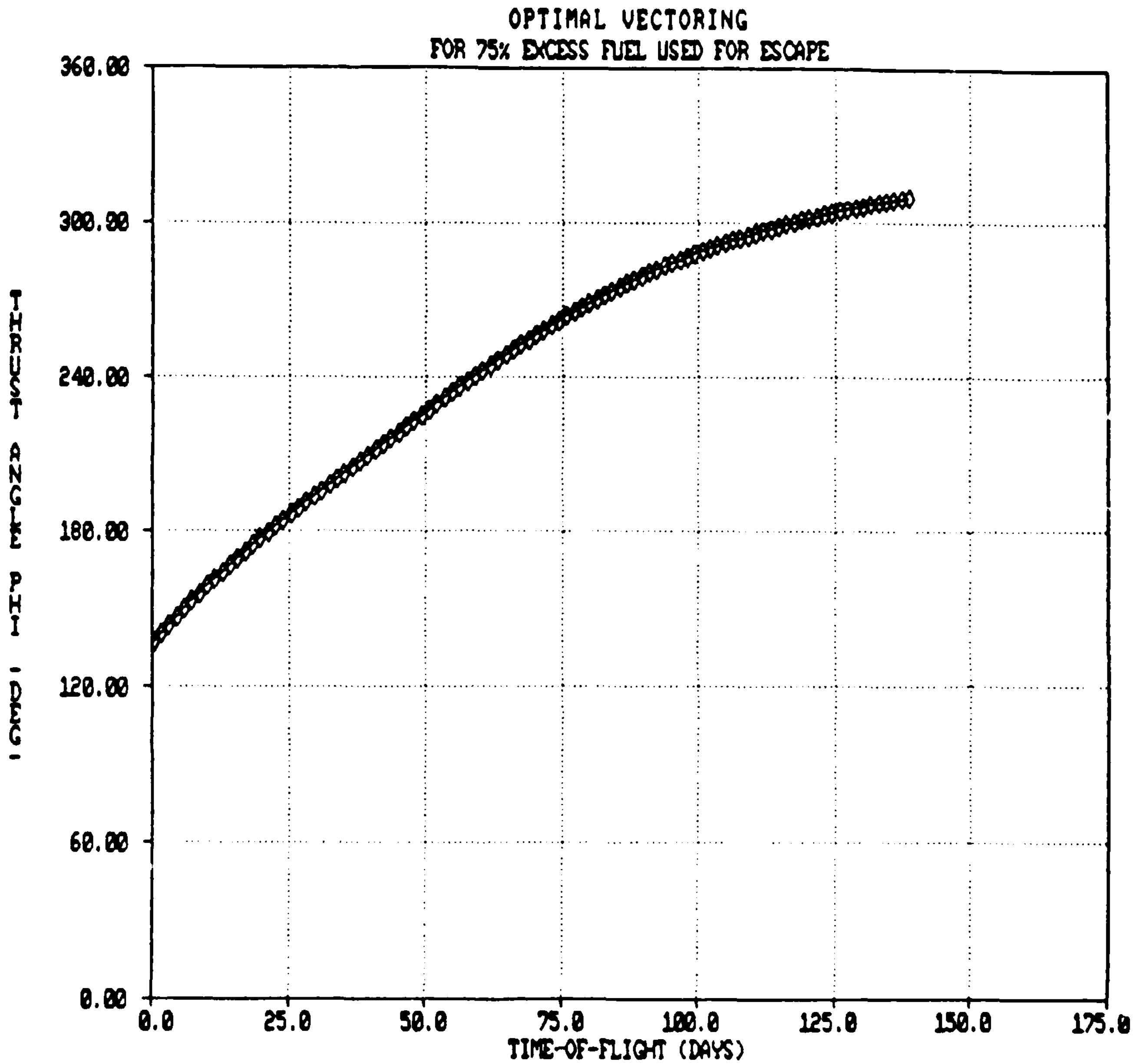


Figure 5.6 Optimal thrust vectoring for 75% excess fuel used for escape

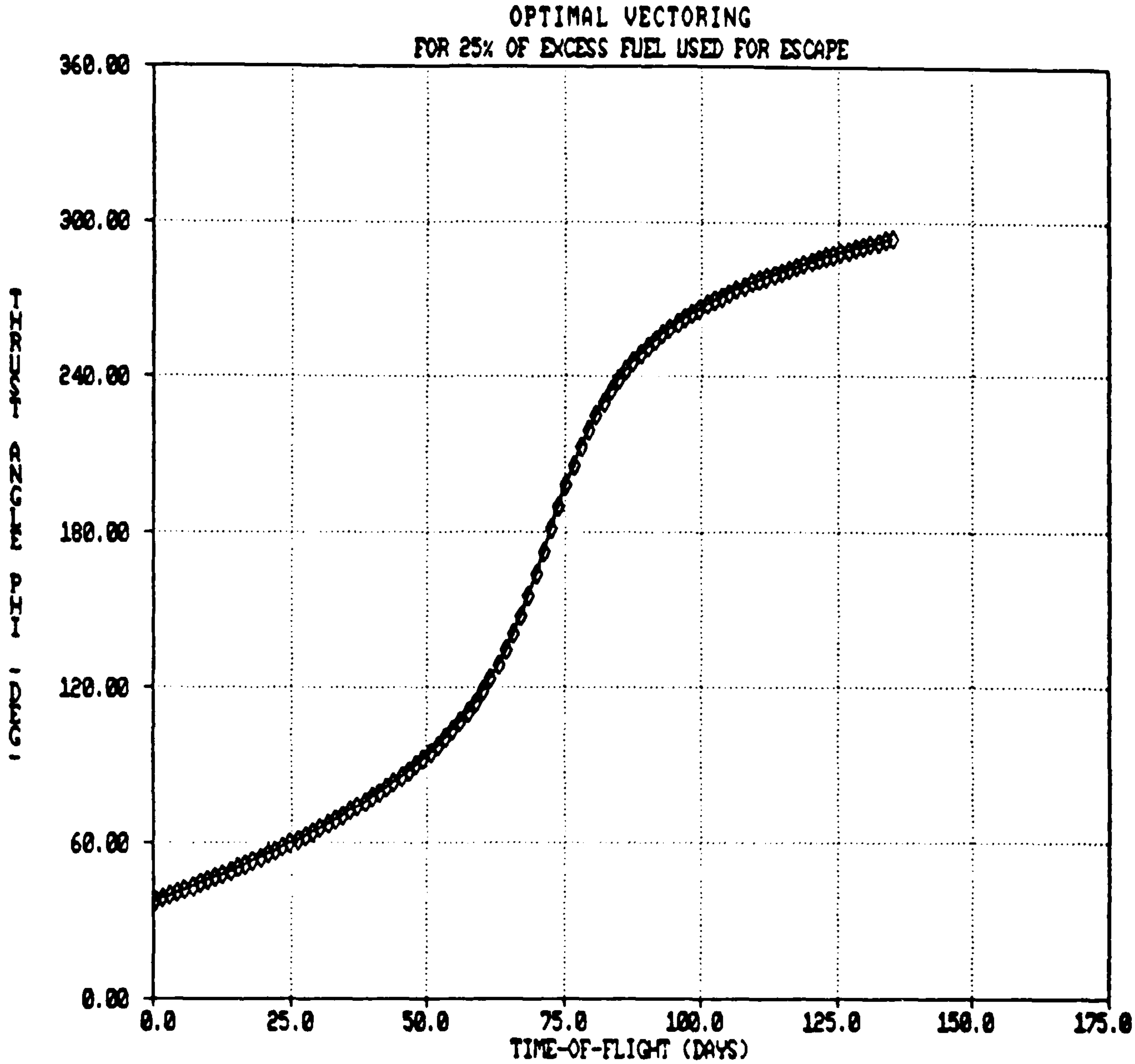


Figure 5.7 Optimal thrust vectoring for 25% excess fuel used for escape

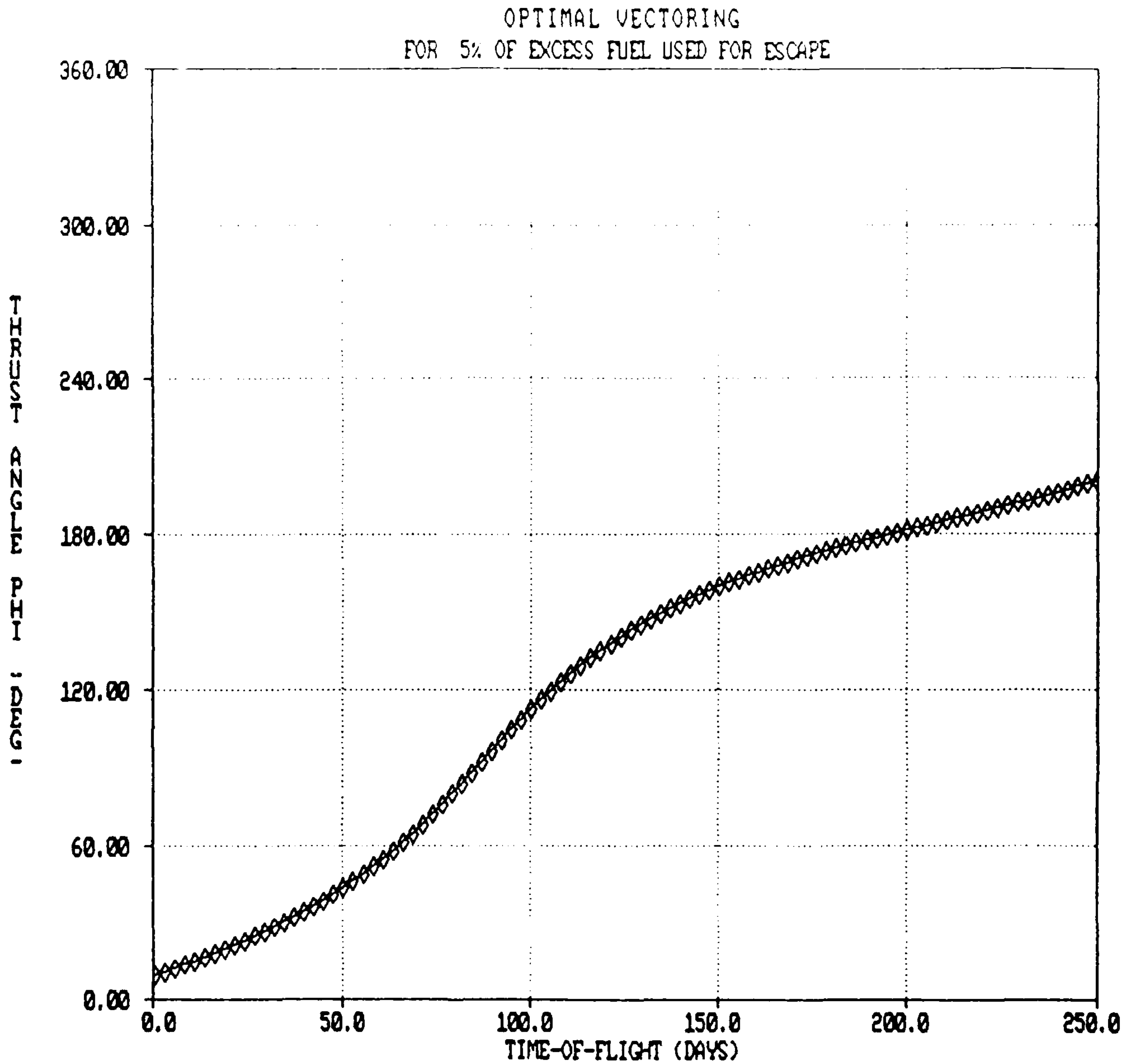


Figure 5.8 Optimal thrust vectoring for 5% excess fuel used for escape

OPTIMAL SIZE OF LOW THRUST PROPULSION UNIT:

Now that the optimal region of injection angle and fuel division has been found, the affect of the relative size of the low thrust unit may be examined by varying μ_T. Using the same assumptions as before, optimizations were repeated for μ_T's ranging from the maximum allowed by the payload ratio and minimum high thrust fuel requirement to that of a small amount of low thrust approaching zero. The results are shown in Figure 5.9. It is evident that if minimum time-of-flight is required, it is best to utilize the available mass for high thrust fuel instead of a low thrust system. However, even as the relative size (and effect) of the electric propulsion system vanishes, the optimal fuel division and the injection angle remain approximately the same. (see figures 5.10 and 5.11)

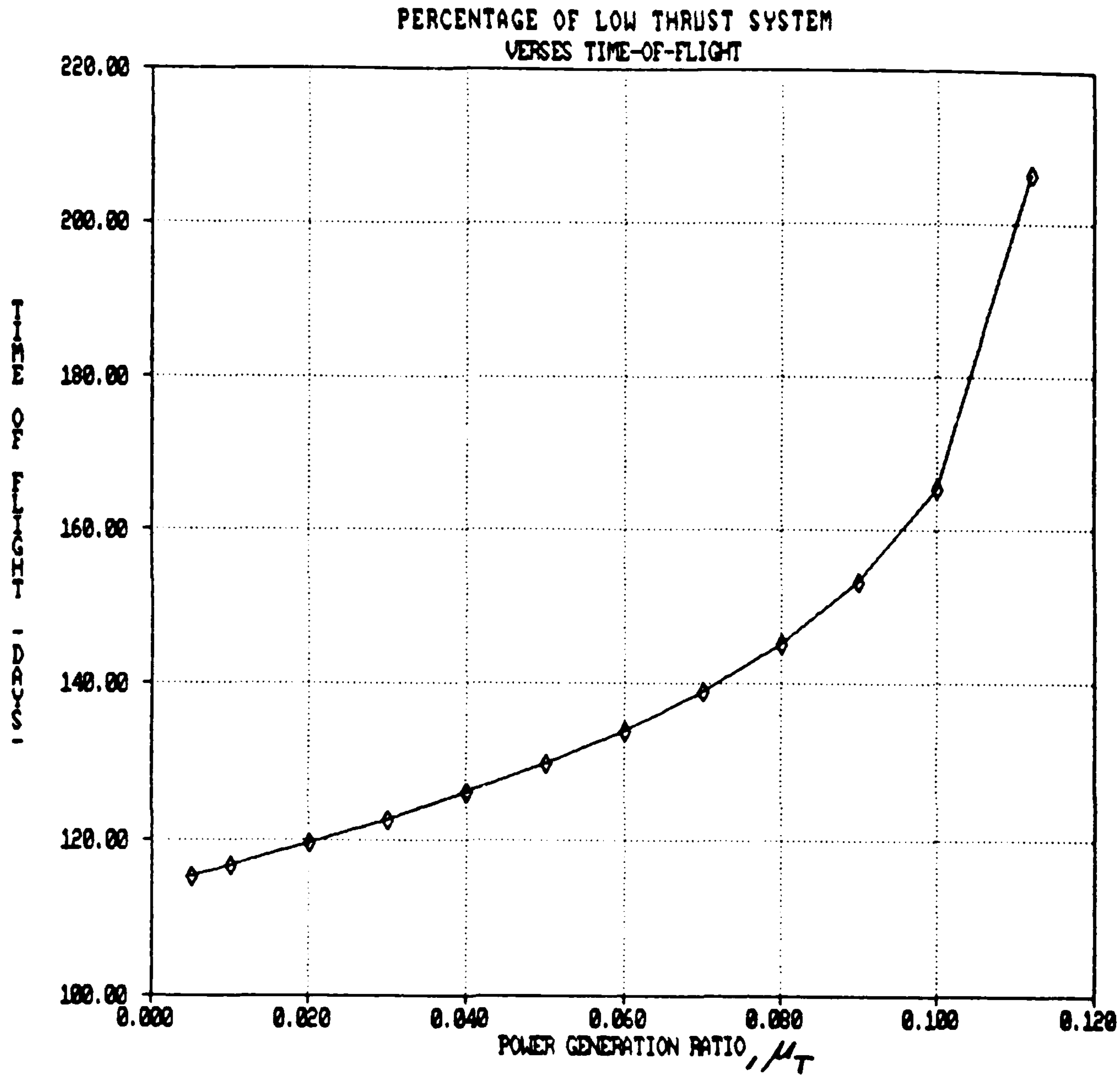


Figure 5.9 Low thrust propulsion system mass ratio verses minimum time-of-flight

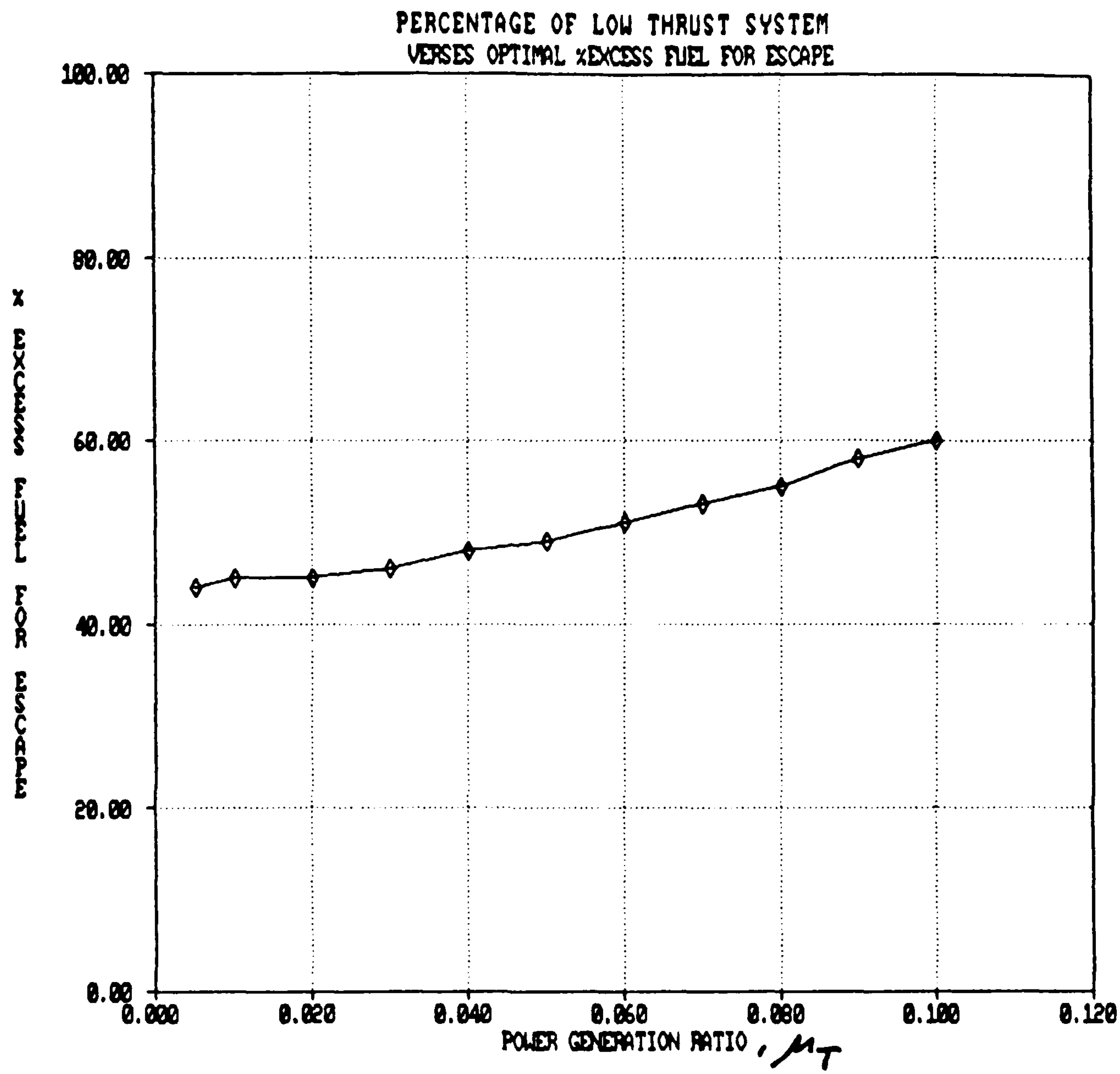


Figure 5.10 Low thrust propulsion system mass ratio verses optimal fuel division

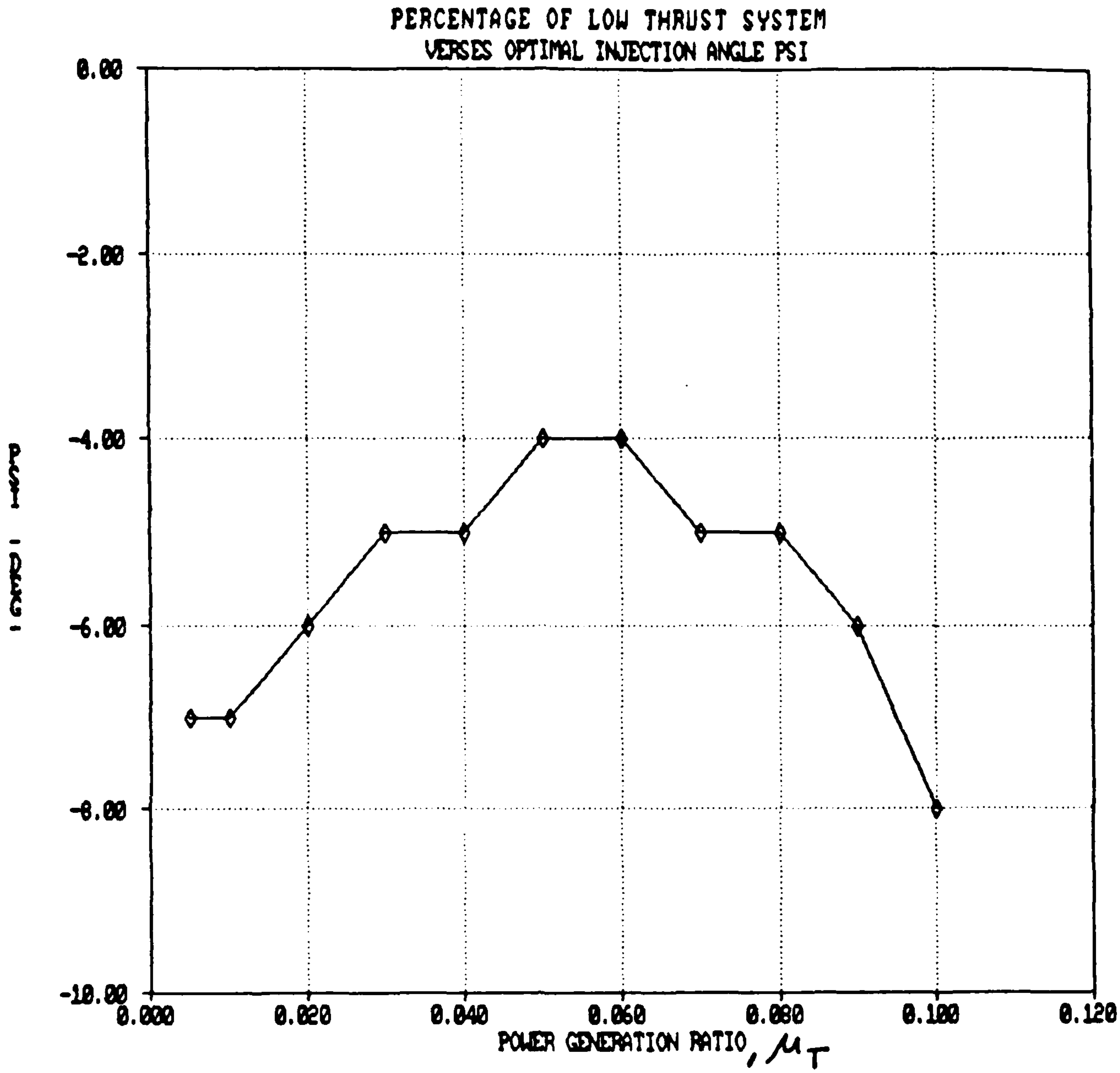


Figure 5.11 Low thrust propulsion system mass ratio verses optimal injection angle

OPTIMIZATION WITH PROJECTED TECHNOLOGIES:

The above optimization will be repeated to determine whether performance is improved, especially when a second reactor for power production is not required. The following parameters are assumed: $\beta = 3.0$, $\alpha = 0.10$, and $\rho = 0.20$. The payload ratio is again $\mu_L = 0.10$. Figure 5.12 shows that although overall flight times are reduced with these parameters, the same basic trend, that of decreased time with decreased μ_T, still exists. Improved spacecraft performance due to the reuse of the nuclear rocket reactor for large amounts of electric power generation does not supply the large expected gain in mass savings. This is due to the small portion (around 20%) of the total required electric power generation mass that consists of the reactor. Consequently, requiring a large, low thrust unit will still envoke a large mass penalty.

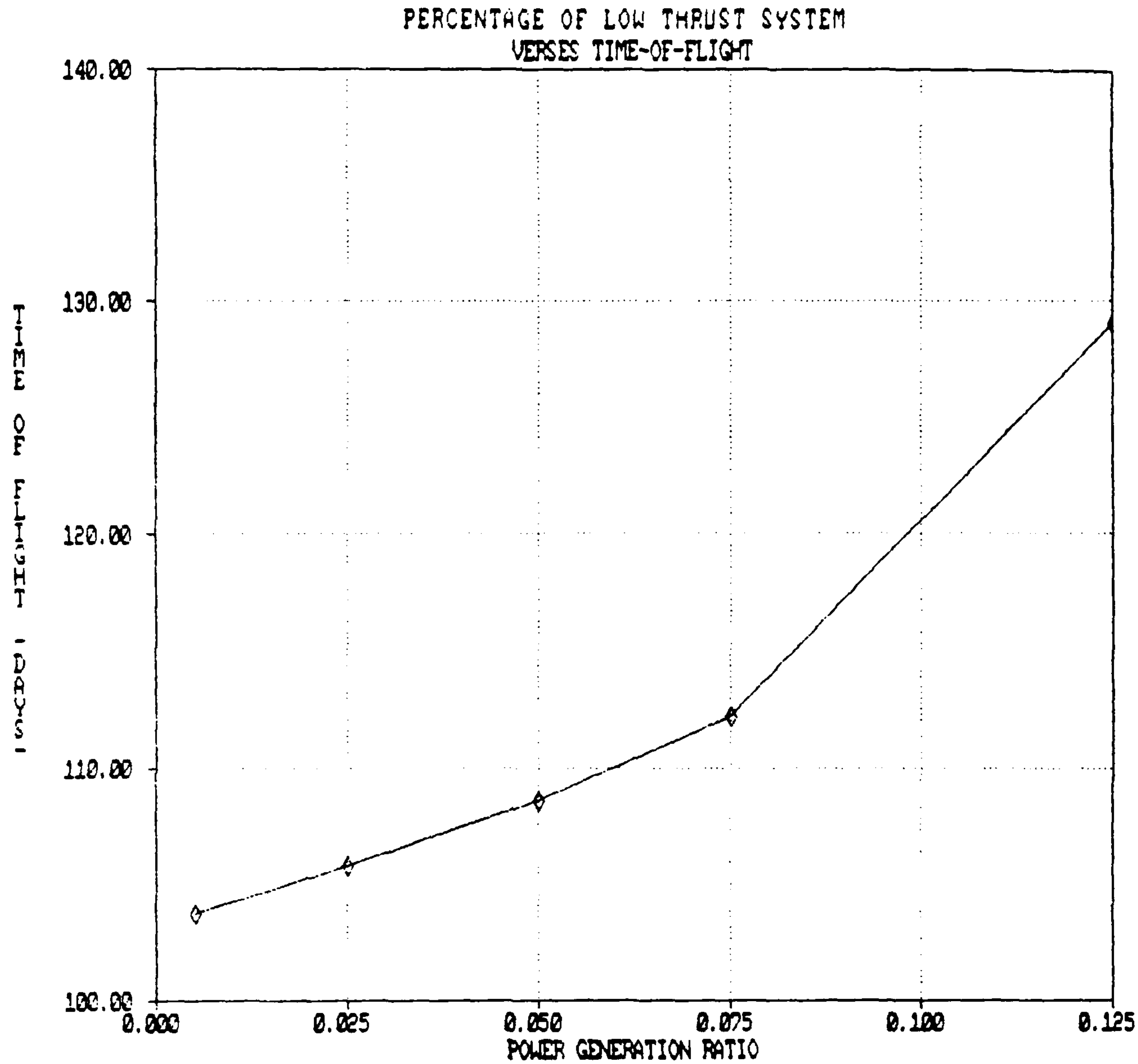


Figure 5.12 Low thrust propulsion system mass ratio verses minimum time-of-flight for advanced technology propulsion systems

CONCLUSIONS AND RECOMMENDATIONS:

We have seen that for the direct transfer from Earth to Mars, the concept of bimodal power propulsion does not provide the short times of flight that a purely nuclear, high thrust system does. This does not, however, discount the use of the rocket reactor for electric power generation to support auxiliary functions of the spacecraft. It has been shown that if excess fuel is utilized in approximately equal amounts for the escape and braking maneuvers the minimum time of flight is found. Consequently, for a mission scenario where the cargo is transported by an electrically propelled vehicle, the passenger spacecraft may use nuclear rockets to complete the transfer to Mars in as short as three months depending upon the rocket technology and the required payload ratio.

The reason the addition of the low thrust system was not found to be beneficial to the minimization of time of flight may be found by noting the real advantage of the low thrust system: higher payload ratios at normally longer flight times. Since a minimum flight time was sought, the low thrust engines, which provide improved thrusting only over time, may not have had enough time to contribute. Perhaps for longer required flight times the combined high/low thrust system would be advantageous.

These optimizations were only completed for a simple, two impulse direct trajectory to Mars. For a return flight that reduces stay time an indirect trajectory taking an inherently

longer flight time would be required. Whether this extended flight time would allow the bimodal low thrust/ high thrust system to give improved performance over the exclusive high thrust system is not known and should be researched. In addition, multiple impulse trajectories have shown improvements over that of the standard two impulse trajectory and may warrant application of the bimodal power propulsion system. Finally, the variation of electric drive thrust and exhaust velocity for optimum flight time [8: p.101-136] could have some affect on the value of the bimodal concept.

APPENDIX A: OPERATING PRINCIPLES OF NUCLEAR BIMODAL PROPULSION

Fission Principles:

To provide thermal energy to heat propellant or to be converted to electrical energy a nuclear reactor very simply converts matter into energy. This occurs by the famous mass-energy relationship

$$\text{Energy} = \text{mass} * (\text{velocity of light})^2.$$

So 1 gram of matter could, in theory, be transformed into 9.0×10^{13} Joules of energy. The great amounts of energy available are obvious. To convert this mass into energy, two general methods may be used: fission and fusion. Fission is the process of splitting an atom to create several atoms and energy. Fusion is the combining of several atoms to create a new atom and energy.

At present, fission is the only method shown to be feasible and in widespread use. After the fission nucleus absorbs the neutron it may emit gamma rays to release the excitation energy gained from the neutron or they may fission or split into two lighter elements and create additional neutrons, gamma rays, and energy (in the form of heat) as well. This process is clearly shown in Figure A.1. The heat produced is from the original elements binding energy and is described by the mass-energy relationship. The normal fission product distribution is shown in Figure A.2. Fission occurs when the excitation energy exceeds the critical energy of an atom.

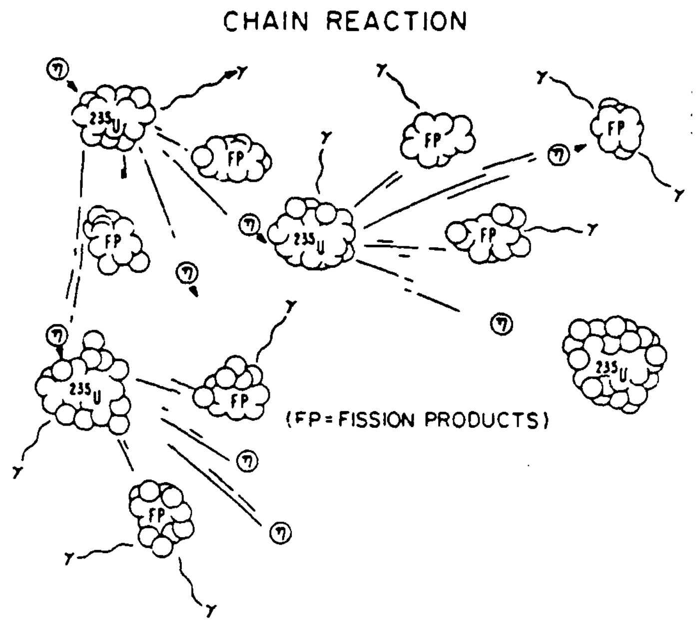


Figure A.1 The fission chain reaction

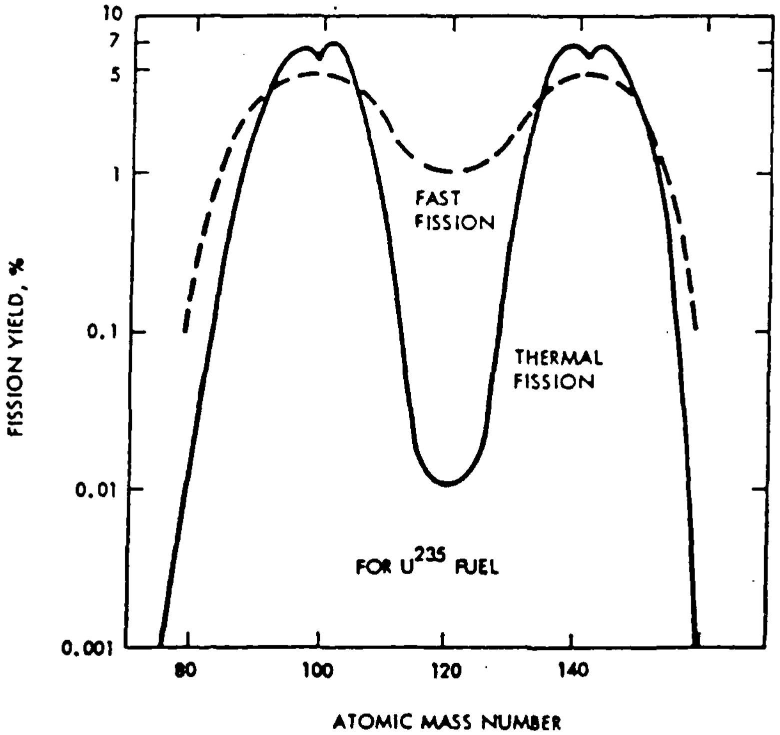


Figure A.2 Basic profile of fission product yields

The critical energy required must be small since the fission energy will be provided by a neutron having fast neutron energies greater than 100 keV. Only atoms with atomic masses above 230 have sufficiently low critical energies. Table A.1 shows the thresholds of such atoms called nuclides. In three cases the critical energy is below zero. In these cases high energy neutrons are not necessarily needed to cause fission, although if high energy neutrons are used, fissions are much more probable and the produced neutrons are also of high energy. The higher probability is directly related to a higher power output of the fission reaction.

Neutronics:

As mentioned, the fission reaction gives off more neutrons. These neutrons may in turn create further fissions. The neutrons given off in fission are termed prompt neutrons. The balance in neutron production is very important since this is what regulates power production. The production of neutrons leads to a chain reaction which stimulates itself; by controlling the production of neutrons the fission process can be made subcritical (the number of fissions will decrease to zero), critical (the number of fissions is constant and balanced), or supercritical (the number of fissions increases each generation).

Table A.1

Neutron fission thresholds of heavy nuclides

Target Nucleus	Compound Nucleus	Fission Threshold (MeV)
Th232	Th233	1.3
U233	U234	< 0
U234	U235	0.4
U235	U236	< 0
U236	U237	0.8
U238	U239	1.2
Np237	Np238	0.4
Pu239	Pu240	< 0

Unfortunately, neutrons are also produced from other sources. The main sources of neutrons are the elements produced by fission which have half lives up to one minute. Neutrons from these sources are termed delayed neutrons since they appear long after the fission generation that produced them. In addition, neutrons can be produced out of sync by reflection, gamma ray/matter interaction, back scatter, and solar flares. All of these nonprompt neutrons, although representing less than one percent of those produced, greatly complicate the control of the fission process.

Neutrons have energies varying from around 0.01 eV to 2 Mev. The probability of fission is increased with the increase of neutron energy as shown by Figure A.3. From 1 eV to 1 keV a resonance region occurs. Operation in such a region would be very difficult and is, therefore, avoided. Hence, nuclear reactors are usually termed 'thermal' (neutron energies $<$ 1 eV) or 'fast' (neutron energies $>$ 100 keV). Due to the low power production of thermal or thermionic reactors only fast nuclear reactors are applicable to nuclear propulsion.

Fissionable materials:

All fission fuels are derived from uranium. Uranium occurs in three natural isotopes (elements differing only in the number

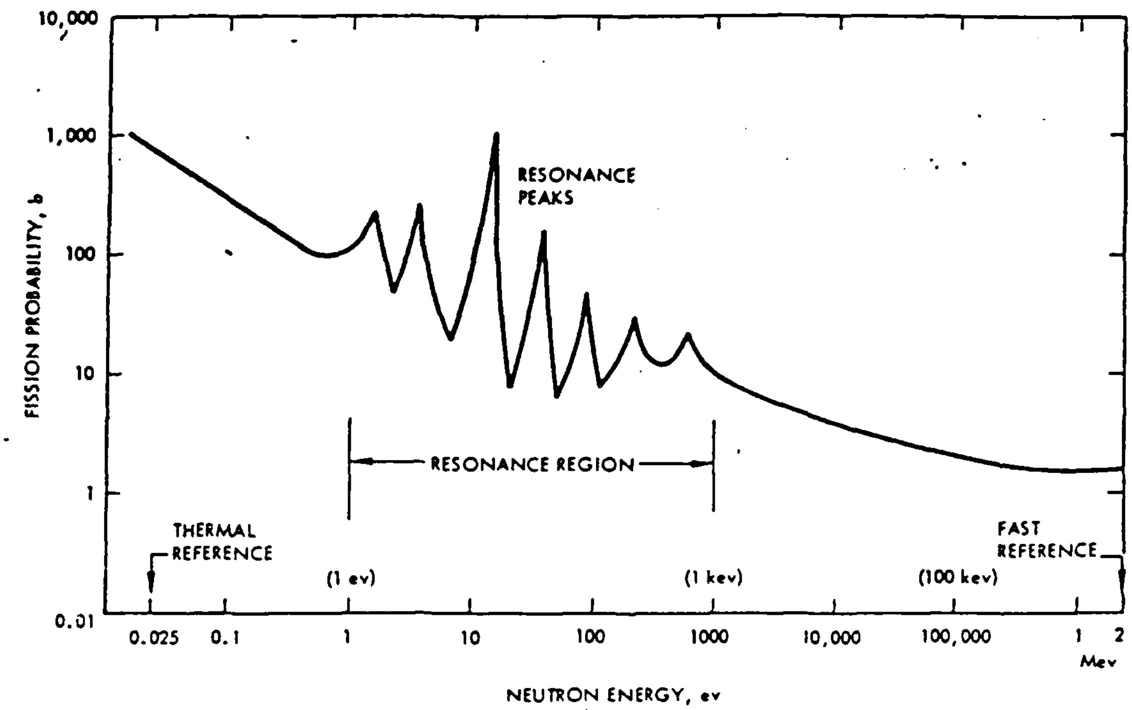


Figure A.3 The generalized fission spectrum (U^{235})

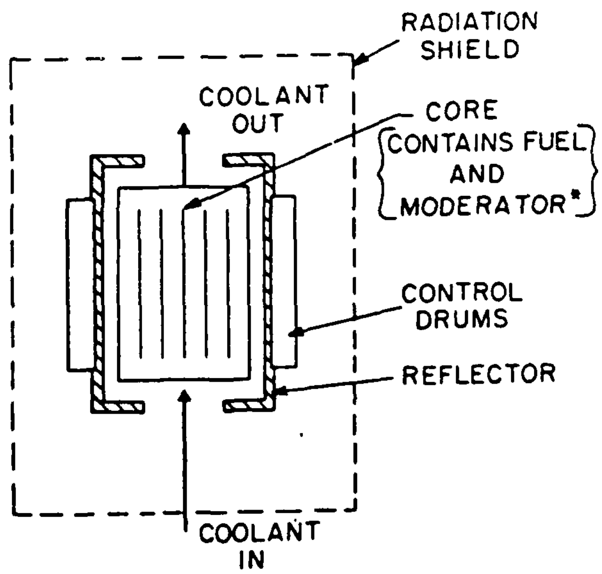


Figure A.4 Generic space nuclear reactor system

of neutrons) in the following abundances:

U^{234} : 0.0056%

U^{235} : 0.7205%

U^{238} :99.2739%

Unfortunately, the only isotope that fissions significantly upon exposure to neutrons is U^{235}, hence expensive techniques are required to separate the U^{235} from natural uranium.

Two other fissionable elements can be artificially produced from more abundant naturally occurring elements. Pu^{239} (plutonium) can be created by neutron bombardment of U^{238} which is very abundant. U^{233} (rarely occurring in nature) can likewise be created by neutron bombardment of Th^{232} (thorium). Each of the three fissionable materials has its own advantages and drawbacks. However, U^{235} is the most fully developed today.

The Fission Reactor:

Many complex technologies must be utilized to contain, control, and exploit the energy of the fission reaction. The basic components of a space reactor are shown in Figure A.4. Each shall be described in turn.

The heart of the reactor is the core. The core contains the fissionable fuel in a matrix to ensure stability. As shall be seen later, the higher the core temperature, the better the open-loop performance. While concepts of higher temperature liquid and gaseous cores have been explored, both are

inappropriate for bimodal use. Only the solid core reactor would be stable at a high temperature (approximately 2600 to 2900 K) for open loop propellant heating and a lower temperature (1100 to 1500 K) for closed loop power generation.

The core is comprised of solid fuel and solid matrix material intermixed. This surrounding matrix material has many important functions: containing and support fuel at high temperatures, protecting fuel from hydrogen attack, permiting advantageous fuel distribution for fission, and allowing for heat transfer to propellant/coolant without thermodynamic reaction with the fuel. Nuclear fuel is basically a very poor structural material and reacts easily with the preferred propellant, hydrogen. However, the matrix material must not absorb neutrons destined for the fuel. Four materials seem candidate to act as a matrix: beryllium, graphite, tungsten, and refractory carbides. In the US nuclear rocket program of the 1960's graphite was used extensively. The matrix material is fabricated to surround individual fuel elements via powder metallurgy. These 'islands' or 'veins' of fission fuel which greatly enhance fuel fissionability are shown in Figure A.5.

The fuel islands are normally not made up exclusively of the fissionable fuel, such as U^{235}, but a compound of the fuel and one or two other elements. This is due to the fissionable fuel low melting point around 1400 K. Candidate fuel compounds are UC, UC_2, UO_2, and UZrC. All of these compounds have melting points above 2500 K.

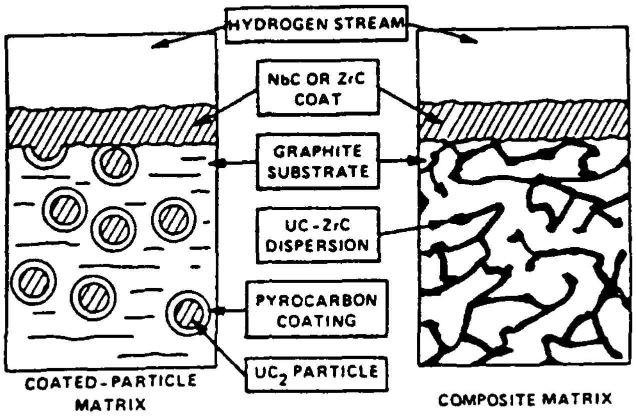


Figure A.5 Sample fuel elements

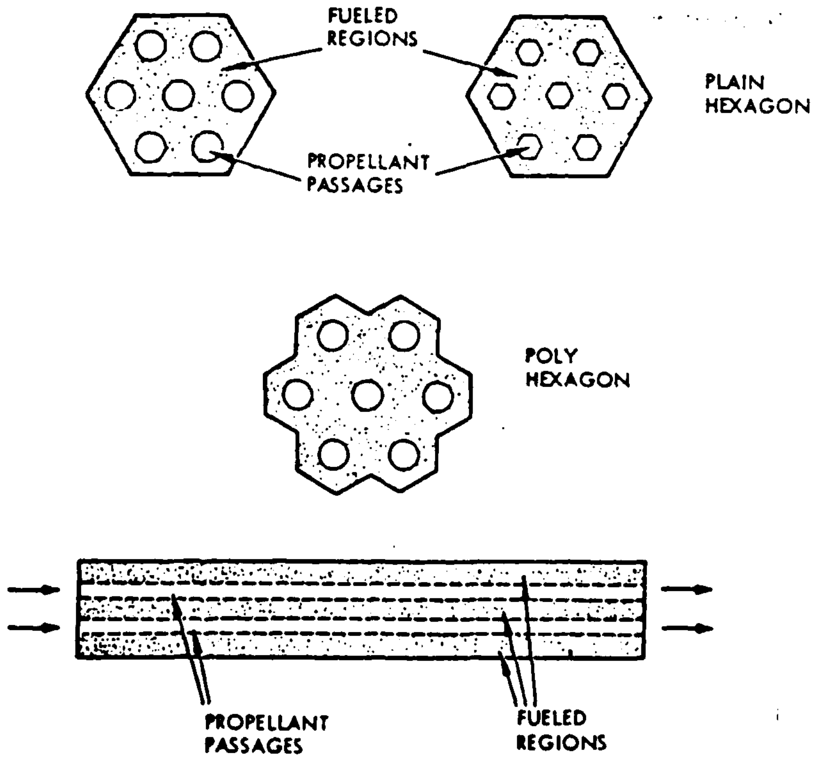


Figure A.6 Fuel/matrix rod configurations

The core matrix is usually fabricated into long slender fuel/matrix rods of hexagonal shape as seen in Figure A.6. These rods contain passages to provide propellant heating and, consequently, must be coated to reduce matrix corrosion with the hydrogen. These rods are then assembled into interlocking units consisting of 7 rods and are termed a fuel module. These modules are in turn combined and wrapped with hoops and placed in a core barrel or cage which structurally contains the core. Due to the greater neutron flux near the center of the core the fuel density is varied radially so as to produce a constant power density profile. (see figure A.7)

Due to the fuel impurities, the core is a fragile part of the reactor. To avoid mechanical stresses and to allow for axial thermal expansion the core is usually only attached to a support plate which doubles as the propellant/coolant flow intake grid. The support plate is in turn attached to the reactor pressure vessel, which acts as the thrust structure transferring the kinetic energy from the nozzle to the space craft. Radial thermal expansion is dampened by high temperature spring seals and hoop springs.

The core must also serve as a heat transfer mechanism allowing all four types of heat transfer (conduction, convection, thermal radiation, and transport) to heat the propellant/coolant and cool the core as shown in Figure A.8.

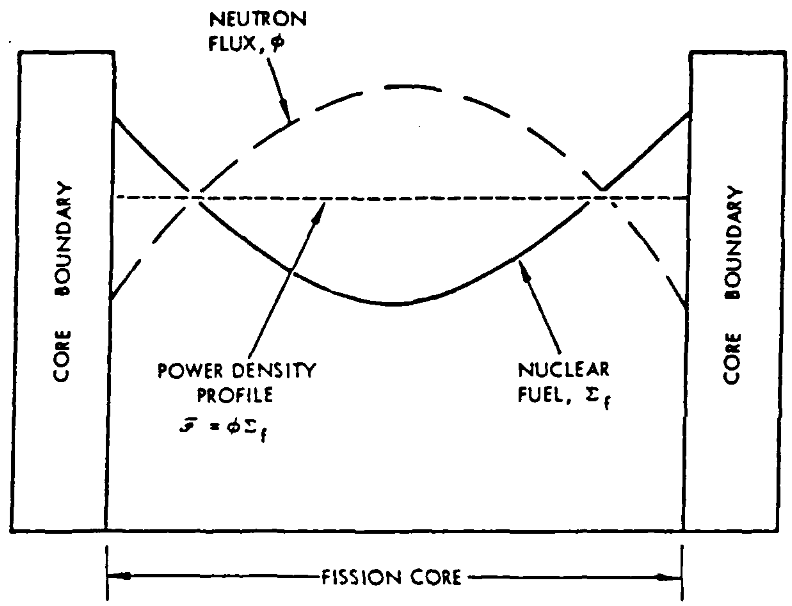


Figure A.7 Fuel variation for tailored power profile

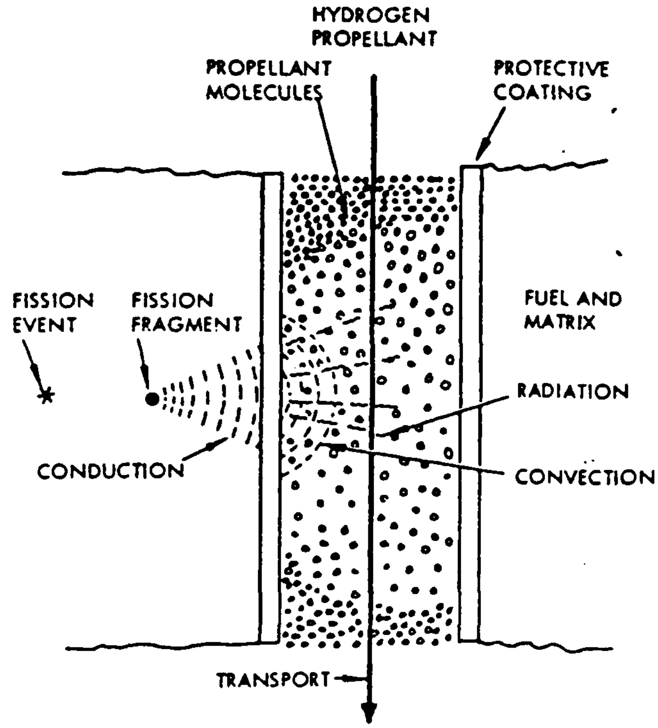


Figure A.8 Basic heat transfer mechanisms

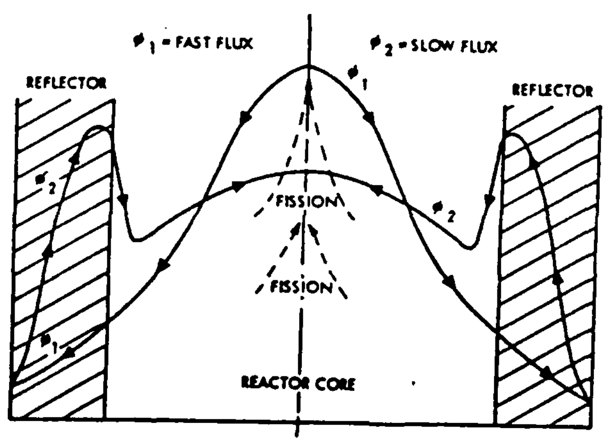


Figure A.9 Effect of two-flux neutrons

The core must be surrounded by a material to reflect the neutrons back into the core so as to keep the fission chain reaction functioning. Called the reflector, this beryllium shield is attached to the pressure vessel and needs to be only 15 to 20 cm thick to reflect 90% of the incident neutrons. The reflector creates a back-flowing slow flux which combines with the normal, nonreflected, fast flux as shown by Figure A.9. As discussed in the previous section, this slow flux does complicate the core criticality. However, the slow flux also has the benefit of stabilizing and dampening the criticality.

The main load-bearing component of the reactor is the pressure vessel. It must focus the load stresses between the nozzle, reactor and the vehicle. Static, dynamic, thermal, vibrational, and acoustic stresses must be controlled. Due to its strength, low density, and high melting point (2000 K) titanium is the preferred material for the pressure vessel.

Controlling the criticallity of the reactor is the function of the control drums. These drums consist of three different materials situated in different areas around the drum. Each material has a different affect on the net neutron flux and hence the decrease or increase in power production. An example control drum cross-section is shown in Figure A.10. From the figure it is evident that by rotating the drums the neutrons produced in fission may either be reflected, absorbed, or both, thereby keeping the fission chain reaction critical.

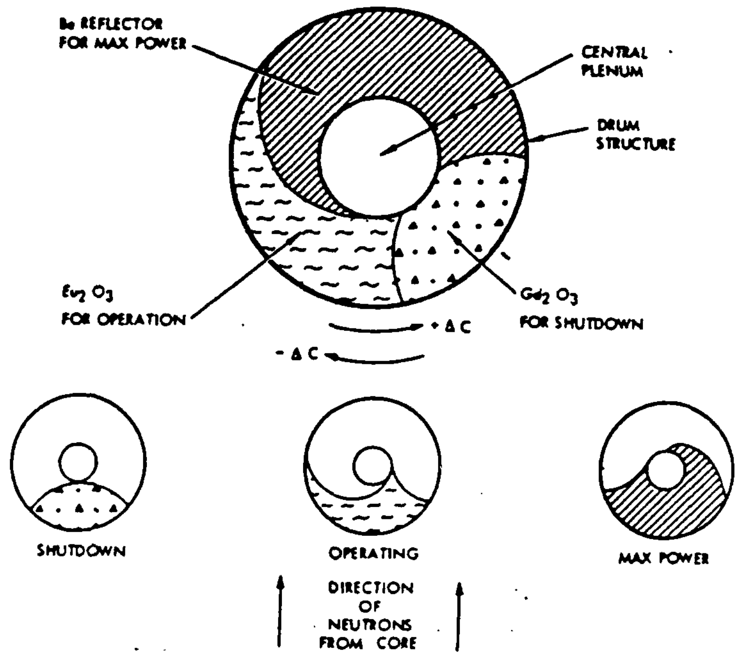


Figure A.10 Sample control drum configuration

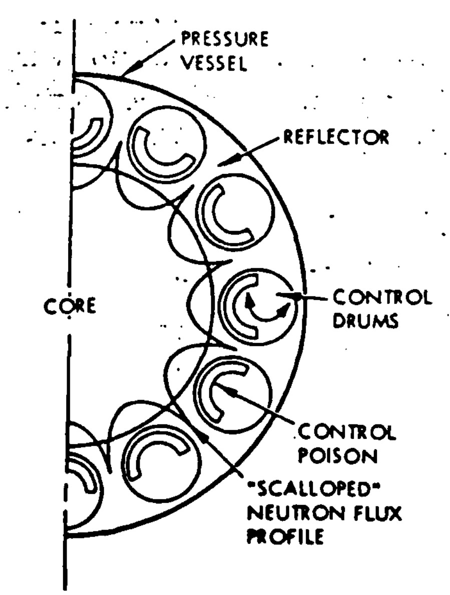


Figure A.11 Control drum pairing

Additionally, there is a higher absorbing material used to shutdown the reaction. Due to the need to move the control drums they must be arranged and operated in pairs on opposing sides of the reactor so as not to change the inertia of the space craft. Several pairs of control drums are usually required as shown by Figure A.11. The drums are usually rotating by a mechanical motor for large adjustments but with pneumatic control for small adjustment verniering.

Often times, the control drums are unable to slow down the fission chain reaction. In these cases neutron poisons are injected into the reactor. These poisons are made of the same neutron absorbers that are found in the control drums. Once the fission reaction is brought under control the poisons may be flushed from the reactor.

The fission reaction gives off radioactivity in the form of neutrons, gamma rays, alpha particles, beta particles and protons. Of these, the neutrons and gamma rays are the most penetrating and potentially damaging to any type of matter, especially living. (see figure A.12) Living beings and electronics suffer the greatest from these radiations. This is because the radiations react with matter by ionizing it, and in the process change the matter's structure and function. Consequently, to avoid the detrimental effects of radiation some form of shielding surronding the reactor must be provided. The geometric coverage of the shielding is dependent upon mission constraints. Different shield

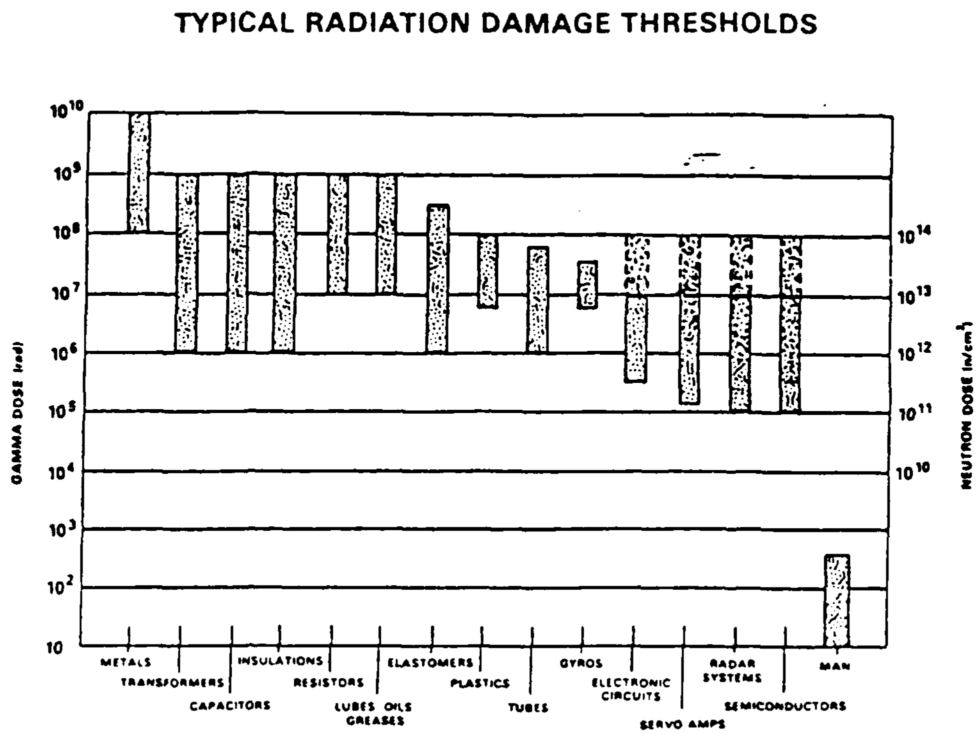


Figure A.12 Typical radiation damage thresholds

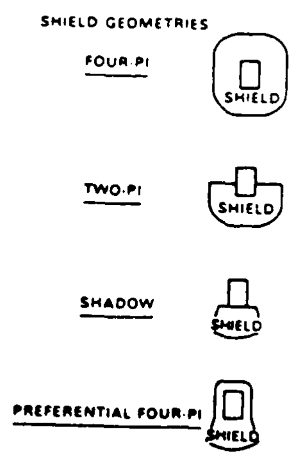


Figure A.13 Various shield design concepts

configurations are shown in Figure A.13. A 4π shield which completely surrounds the reactor is best but is costly in mass and impractical for the open-loop mode; the nozzle end of the reactor cannot geometrically be closed over during operation.

The shadow shield/reactor isolation concept of spacecraft protection is the best compromise between shield mass and crew protection, Figure A.14. Since the radiation dosage falls off with the square of the distance from the reactor the spacecraft may be designed to be long and elongated so as to reduce shield mass and increase the safe 'shadow area'. The favored shielding materials are W (tungsten) and LiH (lithium hydride) for shielding gamma rays and neutrons, respectively. It turns out that neutrons are best attenuated by hydrogen and its compounds. Hence, with the use of hydrogen as a propellant, additional shielding will exist in the form of the propellant tankage. Gamma rays on the other hand are very penetrating and are only attenuated by great amounts of mass. Due to tungsten's high density, it makes for a less voluminous gamma shield. The other forms of radiation, alpha and beta particles, are easily attenuated by the neutron/gamma ray shield. A simple shadow shield utilizing these materials is shown in Figure A.15. Figure A.16 gives a general mass calculation for a 4π shield as a function of power and radial distance.

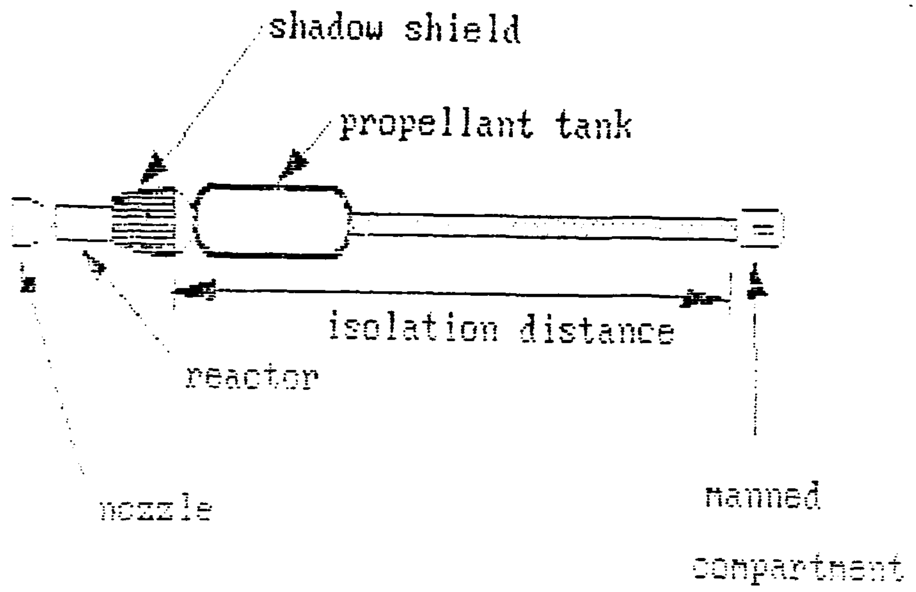


Figure A.14 Application of the shadow shield/reactor isolation concept

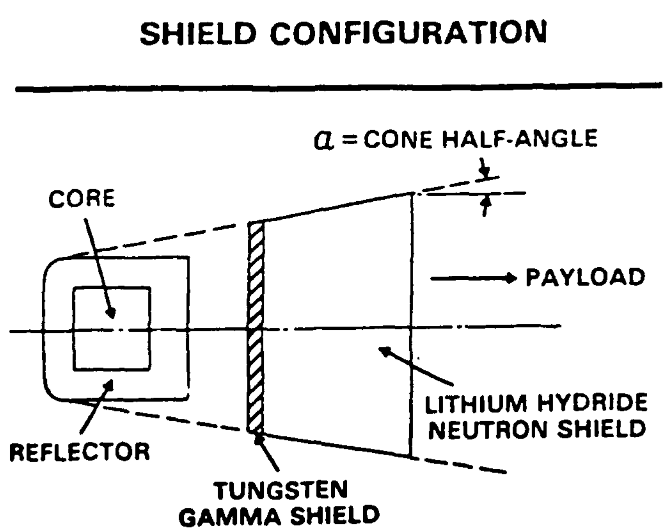


Figure A.15 Typical shadow shield configuration

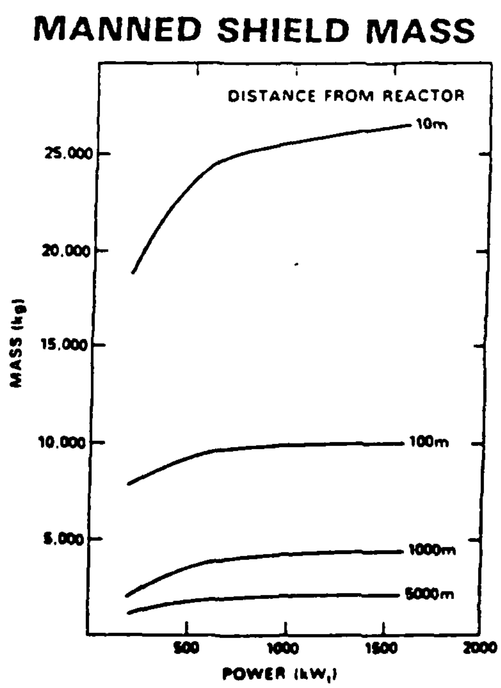


Figure A.16 Typical 4-pi shield mass requirement

The requirement for shielding crew and craft from the dangers of radiation may seem to be a distinct disadvantage for the nuclear propulsion concept. However, outside of the Earth's protective Van Allen belts the sun is a great source of radiation in the form of varying high energy protons and electrons. Consequently, some shielding would be required on any interplanetary mission regardless of power supply. In fact, with proper space craft design, the reactor shields may be used to shield the crew and instruments from solar radiation in addition to the reactor, especially during solar flare activity which increases and decreases on an approximate eleven year cycle.

In 1970, the National Academy of Sciences recommended exposure limits for use by NASA of 25 rem/individual/mission and 400 rem lifetime exposure. The rem is a special radiation dose unit determined by the energy of the radiation and a quality factor describing its penetration.

Two Modes of Operation:

The thermal energy produced by the reactor will be harnessed in two different ways in the bimodal reactor. The open-loop will utilize the reactor energy to heat propellant which, by expansion through a nozzle, will be converted to kinetic energy and, consequently, thrust. The closed-loop will use a coolant to collect thermal energy and, in turn, create electrical energy using familiar turbine techniques. Each mode has special requirements of the reactor and creating a reactor that functions

in both modes is quite a challenge.

Open-Loop Mode:

The open-loop mode uses the reactor's high thermal energy capability to heat a propellant to be expanded. A sample arrangement of an open-loop rocket reactor is given in Figure A.17. The conversion of thermal energy into kinetic energy is described by the equation:

$$V_e = ((2 k R T) / (M (k-1)))^{0.5},$$

where V_e = exhaust velocity

k = ratio of specific heats

R = universal gas constant

T = chamber temperature

M = molecular weight of propellant gas.

Ideal, calorically perfect gases are assumed.

It is obvious the exhaust velocity may be increased (and therefore, so may the specific impulse which is a direct improvement in performance) by either increasing the reactor's chamber temperature T or decreasing the molecular weight of the propellant. By limiting ourselves to solid core reactors we have limited ourselves to a maximum chamber temperature of less than 3000 K. However, the molecular weight may be chosen as small as possible: H_2, molecular hydrogen has a low molecular weight of 2.

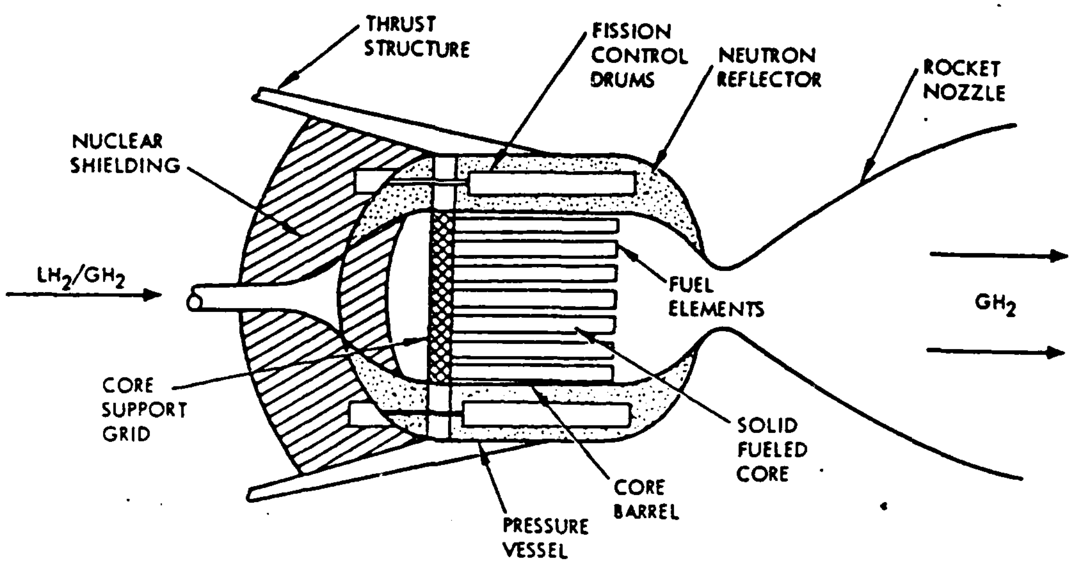


Figure A.17 Introductory arrangement of a rocket reactor

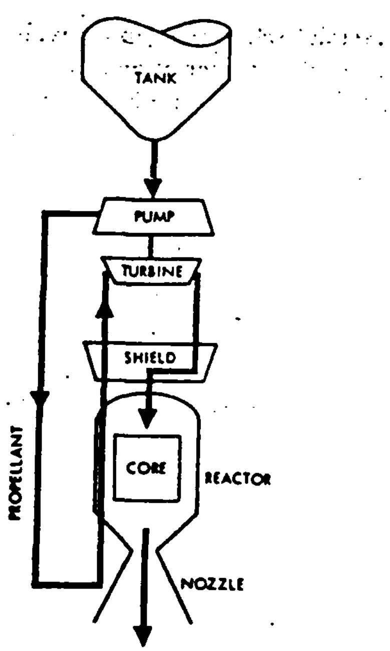


Figure A.18 Sample propellant pumping method

While hydrogen does give the best performance it has some drawbacks as a propellant. Hydrogen is a somewhat reactive chemical and care must be taken to prevent chemical corrosion with the reactor core. Additionally, hydrogen has two storage problems. Hydrogen's gaseous density is only around 0.016 kilograms per cubic meter so the only viable way to store it is to cool it down to liquid form where the density is approximately 48 kilograms per cubic meter. This cryogenic state still requires large tankage structures which bring a mass penalty of about 1.5 times the tankage mass of an equivalent LH_2/LO_2 bipropellant chemical system. Of course these tanks could be jettisoned as the hydrogen was used. Hydrogen must be cooled to around 20 degrees Kelvin to reach its liquid state. Storage for the length of the mission would be required for some of the propellant, adding the mass penalty of adequate thermal blanketing.

Pumping of the hydrogen propellant can be achieved by means similar to those developed for cryogenic chemical rockets. Instead of feeding the hydrogen directly to the core, it is sent to cool the pressure vessel and then travels through a turbine which powers a the pump to feed the hydrogen, Figure A.18. Piping for the hydrogen propellant must be coated to prevent leakage due to hydrogen's small molecular size and its corrosive nature as well as insulated to prevent boil-off. Hydrogen also has many unique properties which makes it more difficult to pump.

Once the hydrogen propellant has passed through the reactor core passages and has gained its thermal energy it is mixed in a chamber, sent through a sonic throat, and expanded through a nozzle. Nozzle cooling is best achieved through film cooling instead of regenerative cooling. (see figure A.19) This is caused by the fact that the temperature limit, hence performance, is derived from the reactor so preheating the propellant will not have the same advantage as that in chemical propulsion.

Although conventional bell nozzles are well developed, they have two drawbacks when used in nuclear propulsion applications. First is the difficulty in vectoring the thrust. Due to the extreme radiation conditions the nozzle must be rigidly attached to the pressure vessel. Consequently, the whole pressure vessel/reactor/nozzle unit would have to be gimbaled, which would require prohibitive actuator loads. The only viable solution would be secondary injection of non-core propellant into the nozzle to create a shock wave and vector the thrust.

However, the conventional nozzle's main drawback is its gathering affect of solar radiation, especially high energy protons and electrons. (see figure A.20) Such incident radiation would be sent to the core where it would greatly disturb the fissions criticallity and complicate reactor control. Consequently, due to the vectoring and incident radiation problems another expansion nozzle concept is required. Fortunately, one

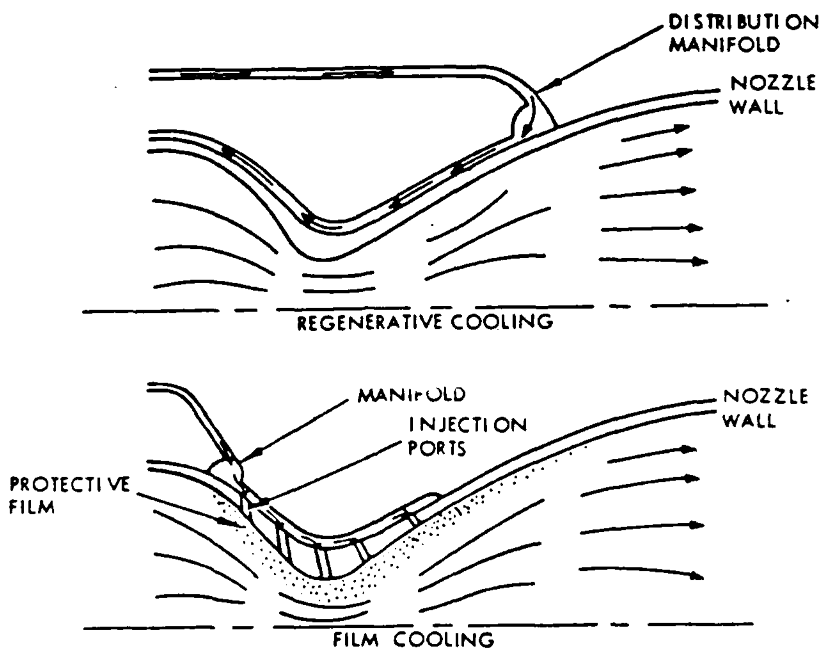


Figure A.19 Nozzle cooling options

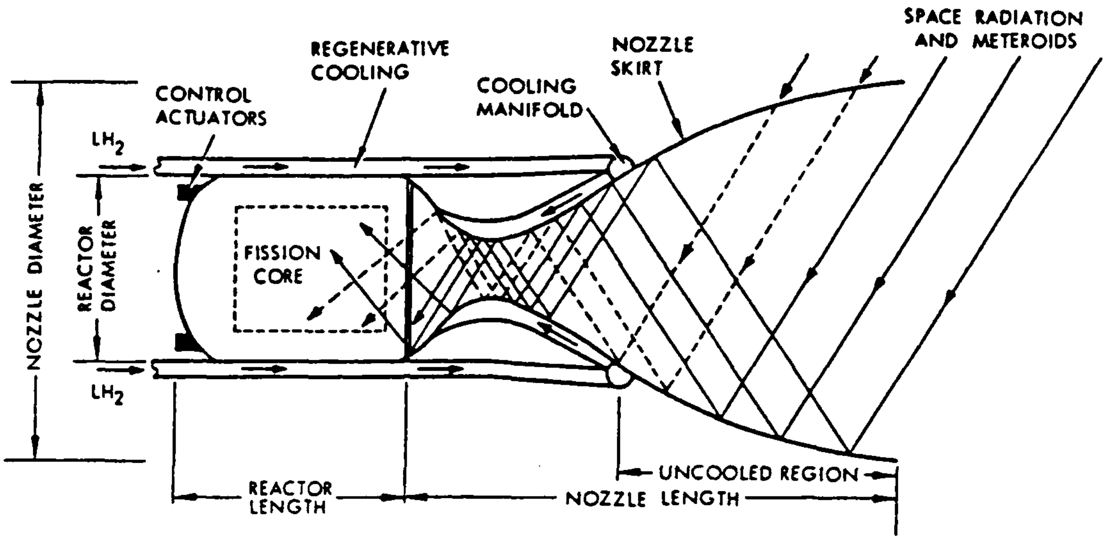


Figure A.20 Nuclear rocket with conventional nozzle

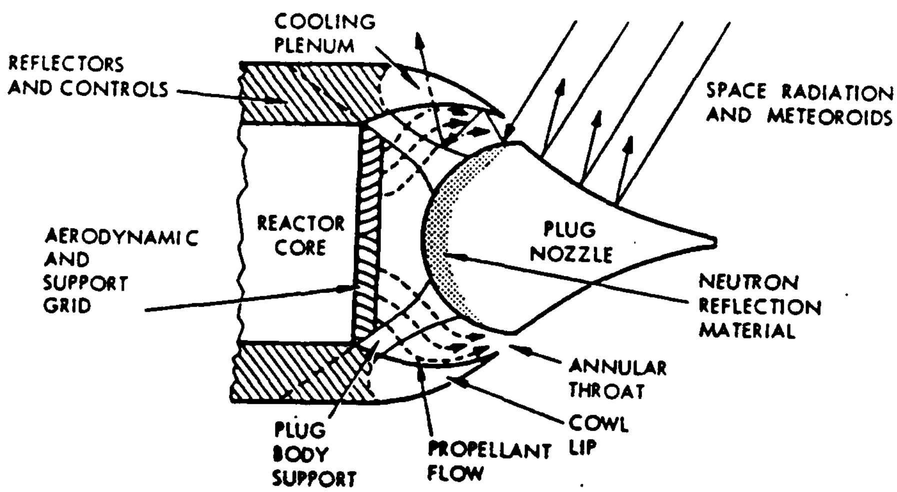


Figure A.21 Nuclear rocket with plug nozzle

has already been developed for use in chemical propulsion: the plug nozzle.

The plug nozzle, shown in Figure A.21, would solve both the vectoring and radiation difficulties. Vectoring is easily achieved by sectioning the 'cowl lip' and controlling the flow in each section to vary the over all thrust direction. The flow at the lip is nearly sonic and expansion of the gases take place along the plug contour just as in a conventional nozzle. The incident solar radiation is easily reflected away from the core by the plug contour. In fact, the presence of the plug reduces the loss of neutrons from the core if the top of base of the plug is coated with reflecting material. This almost completely surrounds the reactor in reflecting material which helps stabilize the fission reaction.

Before analyzing methods of nuclear open-loop propulsion control it is important to look at the equation of thrust:

$$F = dm/dt \; V_e$$

where F = thrust

dm/dt = mass flow rate

V_e = exhaust velocity

It should be recalled that V_e is directly related to the specific impulse (Isp) which is an index of propulsive performance. Thus, thrust may be changed by varying the mass flow rate and/or the exhaust velocity. Recalling that exhaust velocity is a function of core temperature,

$$V_e = ((2 k R T) / (M (k-1)))^{0.5},$$

and keeping all else constant, exhaust velocity may be increased by increasing core temperature. Core temperature is a product of the core's reactive level. Consequently, the exhaust velocity may be increased by merely increasing the fissions in the reactor. The mass flow rate is completely controlled by the pump/turbine speed and capacity. Therefore, unlike chemical rockets where the mixing and flow rates directly affect the chamber temperature and hence the exhaust velocity the nuclear rocket has the exhaust velocity and mass flow rate only indirectly coupled by the heat transfer processes. This important realization would allow variation of the specific impulse, perhaps increasing performance. Such variation could be accomplished by holding the mass flow rate constant and varying the reactor temperature. Of course each nuclear reactor has limits (maximum and minimum) on the reactor temperature and mass flow rate.

Due to the decoupled nature of the reactor temperature and mass flow rate the nuclear open-loop engine may be controlled by varying one or both parameters. A schematic of such a system is shown in Figure A.22. Evident are the temperature/neutronics and

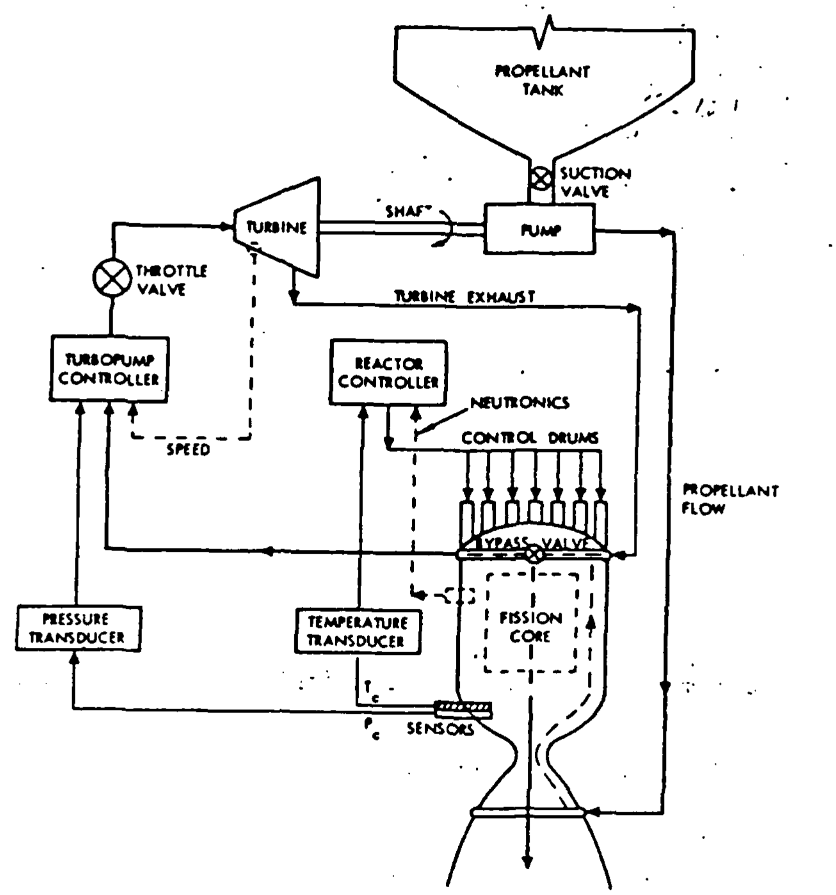


Figure A.22 Schematic of engine control subsystems

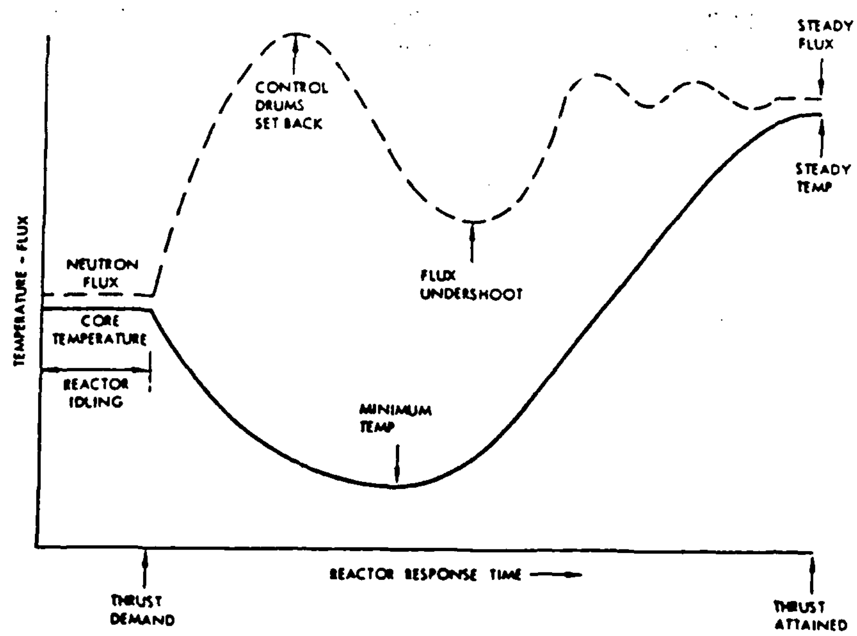


Figure A.23 Core transients during runup

pressure reading instruments which provide data feedback for the reactor and turbopump control, respectively. The heat removed from the reactor (sometimes called the heat generation rate dq/dt) is given by the simple equation:

$$dq/dt = dw/dt \; C_p \; (T_c - T_o)$$

where dw/dt = propellant flow rate

C_p = propellant specific heat

T_c = temperature at core exit (chamber temperature)

T_o = temperature at core inlet

This equation finally shows the coupling of the exhaust velocity and the mass flow rate.

Interaction between these two parameters is clearly shown during a thrust run up in Figure A.23. As hydrogen is pumped into the reactor the temperature falls but the neutrons are better reflected by the control drums causing reactor supercriticallity. To adjust for this supercritical neutron flux, the control drums are turned to absorb these neutrons. As is normal in control theory, overshoot and undershoot occur until the core criticality and temperature stabilize at their desired values. Thus, instability in the control system would have obviously tragic results.

A final requirement of nuclear engine control is the need for cooling the reactor down after its high temperature open-loop thrusting. This may be accomplished by either cooling the reactor with propellant or initiating closed-loop cooling and radiating the waste heat. Both of these methods have disadvantages.

Cooling the reactor with propellant is very effective but is costly in terms of required propellant. Using the closed-loop coolant system would require high temperature capabilities not normally needed in the lower temperature closed-loop operation.

Closed-Loop Operation:

The reactor's closed-loop function is that of generating electrical power. This power is for use mainly by the ion propulsion units but may also be used by the spacecraft itself for controls, housekeeping and life support functions.

Normal, single mode space reactors which produce only electrical energy are similar to the open-looped ones described in the previous section. (A sample of a closed-loop reactor is given in Figure A.24.) However, they differ in several aspects:

1. Closed-loop reactors have much lower operating core temperatures: around 1000 K as compared to open-loop temperatures approaching 3000 K. This is mainly due to the material limits of the power generating equipment.

2. Instead of propellant, coolant is cycled through the reactor. This coolant is usually a liquid metal such as sodium potassium (NaK) or silicon germanium (SiGe) due to its combined thermal convection and conduction rates.

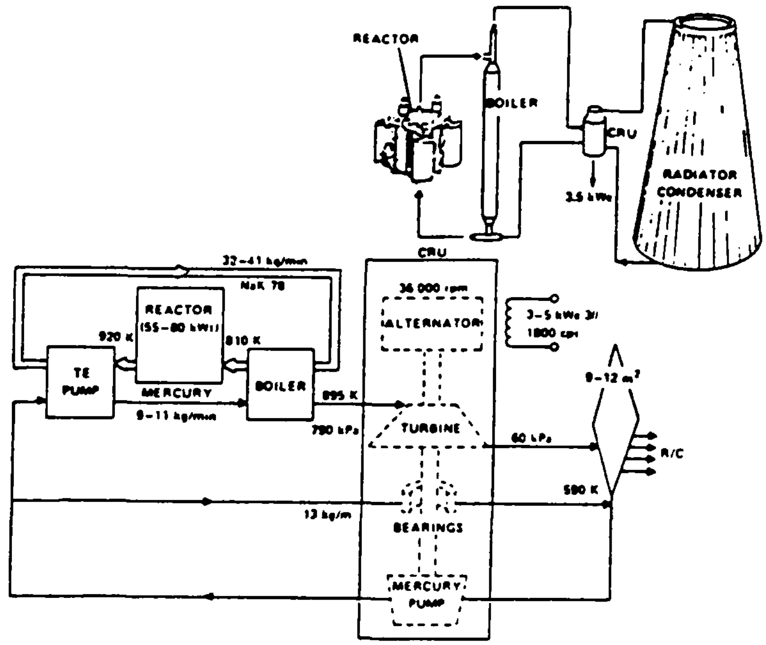


Figure A.24 SNAP-2 mercury Rankine system schematic

MODES
- HIGH POWER ROCKET MODE OF 365 MW, 10 h LIFE, 2600-2660 K PROPELLANT TEMPERATURE, NEGLIGIBLE FUEL BURNUP
- LOW POWER ELECTRICAL MODE OF 10-25 kW(e), 1 MW(t), ORGANIC RANKINE CYCLE DRIVEN BY THERMAL ENERGY FROM STRUCTURAL SUPPORT SYSTEM, USES PROPULSION MODULE SURFACE TO SUPPORT RADIATOR UP TO 10 kW(e)

ENGINE MODIFICATIONS
- STRUCTURAL SUPPORT SYSTEM ISOLATION VALVES FOR LOW-POWER
- CHANGE REACTOR DOME AND TIE TUBE CORE SUPPORT PLATE LINES FOR WIDER TEMPERATURE RANGE OPERATION TO STAINLESS STEEL FROM Al, AND ACTUATOR WINDINGS TO CERAMIC FROM POLYIMIDE

LIFETIME
- STUDY SHOWED TWO YEARS LIFE WILL NOT SIGNIFICANTLY AFFECT REACTIVITY CONTROL MARGIN

*STUDIED IN EARLY 1970's BY J. ALTSEIMER, L. A. BOOTH, "THE NUCLEAR ROCKET ENERGY CENTER CONCEPT" LA-DC-72-1262, 1972, BASED ON IDEAS OF JOHN BEVERIDGE

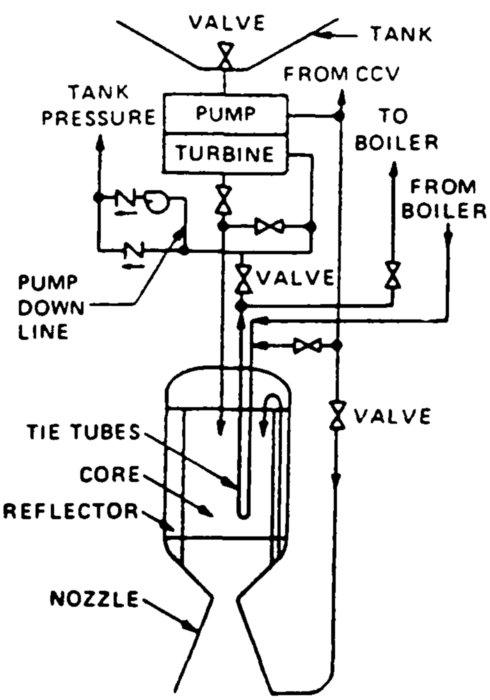


Figure A.25 Sample dual mode nuclear rocket

3. The heat of the liquid metal coolant is transferred to a secondary coolant usually through a heat exchanger. This prevents radioactive coolant from being exposed to the power generating apparatus. Typical secondary coolants are mercury (Hg), Xenon (Xe), and helium (He).

4. The secondary coolant is used in an energy conversion cycle such as the ideal Rankine cycle, the ideal Brayton cycle, or the ideal Stirling cycle. Each cycle has its own merits and disadvantages. Basically, heat energy is converted to kinetic energy which is finally converted to electrical energy for use by the spacecraft. Recalling the laws of thermodynamics some of the heat is left over from the cycle and must be radiated into space.

With these differences in mind, some observations may be made as to how to adapt a open-loop reactor into a bimodal reactor. Due to the presence of the hydrogen propellant in the open-loop cycle it is perhaps best that hydrogen also function as the primary coolant replacing the normal liquid metal. The secondary coolant and the energy conversion cycles could still be used but with perhaps some modifications. The reactor's temperature and consequently, reactivity would have to be reduced. This would call for adjusting the reactor nucleonics from open-loop power mode to closed-loop power mode. Such adjustments could come in the form of more absorbing material and additional controls. Consequently, it seems that bimodal reactor operation is possible. However, such reactors have yet to be developed and present the biggest challenge to the success of the bimodal nuclear propulsion

concept. A sample schematic of a bimodal 'dual-mode' nuclear rocket is presented in Figure A.25.

Once the electrical power is created it may be used for any and all spacecraft requirements. However, from the propulsion point of view, a major portion of the power will be used for the ion engines.

The theory and performance of ion engines is quite extensive and will only be briefly covered here. For further information see the references in the bibliography.

The ion engine functions by 1.) ionizing a propellant and 2.) accelerating that propellant to high velocities using an electrical potential difference. Many ion engines have been developed to date but few have been actually used by spacecraft.

Electrical energy is required to both ionize and accelerate the propellant. The exhaust velocity attained by the ion engine is a function of the potential difference (Voltage) and the ion charge/mass ratio such that,

$$\text{exhaust velocity} = (2 * \text{charge/mass} * \text{voltage}),$$

the mass flow rate is defined by

$$\text{mass flow rate} = \text{mass/charge} * \text{electrical current}.$$

Consequently, the thrust of an ion engine is

$$\text{thrust} = \text{mass flow rate} * \text{exhaust velocity}$$

$$\text{thrust} = (2 * \text{mass/charge} * \text{current} * \text{voltage})^{0.5}.$$

Unlike the thermal nuclear engine, an increase in exhaust

velocity is usually not required due to ion engines readily available high exhausts. Such high velocities on the order of 100000 m/sec are considerably higher than optimum for most interplanetary missions. Consequently, it is desirable to decrease the exhaust velocity by choosing a larger mass-to-charge ration and, in turn, heavier propellants. Propellants are also chosen on their ease of ionization. Sample propellants in use are cesium (Cs), mercury (Hg), and xenon (Xe).

From mission requirements a thrust and desired exhaust velocity may be determined. Consequently, the electrical input power required is merely

$$W_{in} = \text{voltage} * \text{current} * \text{efficiency factor} + \text{ionization power}$$

CONCLUSIONS:

The bimodal nuclear propulsion system is a highly complex system relying upon all engineering disciplines. And while such bimodal systems have yet to be built the technology of nuclear open-loop rockets, nuclear closed-loop space power, and ion engines has been well developed. At the very least, the bimodal system could provide electrical power to support the spacecraft and consequently, only provide high thrust propulsion. While other obstacles to interplanetary flight remain, such as microgravity effects upon man and funding, the bimodal nuclear propulsion system could provide a reliable, timely, and reusable method for manned exploration of our solar system's planets.

Appendix A: Bibliography

Angelo, J., and D. Buden:"Space Nuclear Power," Orbit Book Company, Inc., Malabar, FL, 1985.

Bate, R., D. Mueller, and J. White: "Fundamentals of Astrodynamics," Dover Publications, Inc., New York, 1971.

Berman, A.: "The Physical Principles of Astronautics," John Wiley & Sons, New York, 1961.

Bussard, R.W., and R.D. DeLauer:"Fundamentals of Nuclear Flight," McGraw-Hill Book Company, New York, 1965.

Crouch, H.:"Nuclear Space Propulsion," Astronuclear Press, Granada Hills, Calif., 1965.

Ivanov, A., V.D. Kolganov, V.A. Favchcock, V. Sarjonev: Nuclear Power Propulsion System Concept For A Manned Martian Mission, submitted at the 1989 Space Power Systems Symposium Albuquerque, N.M.

Jahn, R.: "Physics of Electric Propulsion," McGraw-Hill Book Company, New York, 1968.

Stuhlinger, E.: "Ion Propulsion for Space Flight," McGraw-Hill Book Company, New York, 1964.

Sutton, G.: "Rocket Propulsion Elements," John Wiley & Sons, New York, 1986.

APPENDIX B: PROGRAMS

```
      PROGRAM MINTIME
C     PERFORMS INTEGRATION OF ORBIT AND DETERMINES PHI(T)
C     THRUST ANGLING FUNCTION SUCH THAT FINAL CONDITIONS
C     ARE SATISFIED AND TRANSFER TIME IS MINIMIZED
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /HAM/ T,X(42,4),F(42,4),ERREST(42),N,H,MODE
      DOUBLE PRECISION XIC(6),B(3,6),DELX(3),DTOT(3),CPRIME(3,3)
      DOUBLE PRECISION DELL(3),PHIMATRIX(6,6),C(3,6),FUDGE(3)
      EQUIVALENCE(DELX,DELL)
C
C     READ IN INTIAL CONDITIONS
      XIC(1)=1.0D0
      XIC(2)=0.0D00
      XIC(3)=1.0D0
      WRITE(*,*)'INPUT LAMBDA R GUESS'
      READ(*,*)XIC(4)
      WRITE(*,*)'INPUT LAMBDA U GUESS'
      READ(*,*)XIC(5)
      WRITE(*,*)'INPUT LAMBDA V GUESS'
      READ(*,*)XIC(6)
      WRITE(*,*)'INPUT FINAL DESIRED CIRC. ORBIT RADIUS'
      READ(*,*)RM
C
C     SPECIFY FINAL TIME
      TF=3.323D+00
      TFOLD=TF-TF/10.D0
      RFOLD=RM-RM/10.D0
C
C     READ IN MAX ITERATIONS, # OF STEPS
      MAXIT= 50
      WRITE(*,*)'INPUT NUMBER OF STEPS'
      READ(*,*)NSTEP
C
C          SET UP HAMING
      N=42
      T=0.0D0
      MODE=1
      DSTEP=NSTEP
      H=TF/DSTEP
C
C     LOOP ON ITERATIONS
      DO 190 II=1,3
         DTOT(II)=0.00D+00
         DELX(II)=0.00D+00
 190     DELL(II)=0.00D+00
C
      OPEN (9,FILE = 'THEONE.OUT')
      WRITE(*,*)'FILES OPENED'
```

```
C
C     MAIN PROGRAM LOOP
      DO 2000 ITER=1,100
       NSTEP=1000
       DSTEP=NSTEP
       H=TF/DSTEP
       RITE(*,*)'H= ',H
       OPEN (8,FILE = 'THEONE.PRN')
       WRITE(9,*)'ITER=',ITER
C      INITIALIZE ERROR VECTORS
        DO 195 J=1,3
          DELX(J)=0.00D+00
 195      DELL(J)=0.00D+00
C
C       SET UP IC'S FOR ORBIT, PHI INTEGRATION
C       INITIAL X(1-3,NXT) NEVER CHANGES, CHANGE LAGRANGE FACTORS
          DO 200 II=1,3
 200          X(II,1)=XIC(II)
          DO 205 II=1,3
              X(II+3,1)=XIC(II+3)+DTOT(II)
 205          WRITE(*,*)'NEW IC X( )=',II+3,X(II+3,1)
C
         IF (IRESPOND.EQ.1) THEN
             WRITE(*,*)'GO(1),STOP(2),OR NEVER STOP(3)'
             READ(*,*)IRESPOND
             IF (IRESPOND.EQ.2) STOP
         END IF
C       INTIALIZE PHI MATRIX
          DO 210 II=7,42
 210          X(II,1)=0.00D0
          DO 220 II=7,42,7
 220          X(II,1)=1.00D+00
C
C        RESET INITIAL TIME
              T=0.00D0
C
C        INTEGRATE ORBIT
          NXT=0
          DO 225 II=4,6
 225          WRITE(9,*)'X( )=',II,X(II,1)
          WRITE(*,*)'GO TO HAMING 1'
          MODE=1
          CALL HAMING(NXT)
           IF(NXT.EQ.0) THEN
               WRITE(*,*) 'HAMING FAILED!'
               GO TO 9999
           ENDIF
C
C         MAIN HAMING INTEGRATION LOOP
           JJJ=10
           DO 300 I=1,1000
              MODE=1
              CALL HAMING(NXT)
               FPHI=DATAN2(X(5,NXT),X(6,NXT))
                JJJ = JJJ + 1
```

```
                IF (JJJ.GE.10) THEN
                     WRITE(8,*)T*58.132821D00,FPHI*57.29D00
                    JJJ=0
                END IF
 300       CONTINUE
C
C     ORBIT INTEGRATED
           WRITE(9,*) 'ORBIT INTEGRATED'
C
C     CALCULATE ERROR VECTOR
           XMU=1.0D0
           DELX(1)=X(2,NXT)
           DELX(2)=X(3,NXT)-DSQRT(XMU/X(1,NXT))
           DELX(3)=X(4,NXT)-1.0D0-X(6,NXT)*DSQRT(XMU)/
     1             (2.D0*X(1,NXT)**(1.5D0))
C          TRANSFORM PHI TO NORMAL 6X6
           ICOUNT=0
           DO 400 ICOL=1,6
              ICOUNT=ICOUNT + 6
             DO 400 IROW=1,6
              PHIMATRIX(IROW,ICOL)=X(ICOUNT+IROW,NXT)
 400        CONTINUE
             WRITE(9,*)'THE PHI MATRIX'
             WRITE(9,*)((PHIMATRIX(I,J),J=1,6),I=1,6)
C
C         CALCULATE THE B MATRIX
           DO 410 I=1,3
            DO 410 J=1,6
 410           B(I,J)=0.D0
           B(1,2)=1.D0
           B(2,1)=.5D0*DSQRT(XMU)/(X(1,NXT)**1.5D0)
           B(2,3)=1.D0
           B(3,1)=1.5D0*X(6,NXT)*DSQRT(XMU)/(2.D0*(X(1,NXT)**2.5D0))
           B(3,4)=1.0D0
           B(3,6)=-DSQRT(XMU)/(2.D0*(X(1,NXT)**1.5D0))
C
C         OUTPUT B MATRIX
          WRITE(9,*)' THE B MATRIX'
          WRITE(9,*)((B(I,J),J=1,6),I=1,3)
C          MULITIPLY C=B*PHI
           DO 460 II=1,3
            DO 460 JJ=1,6
              C(II,JJ)=0.0D0
              DO 450 KK=1,6
               C(II,JJ)=C(II,JJ)+B(II,KK)*PHIMATRIX(KK,JJ)
 450          CONTINUE
 460       CONTINUE
C
          WRITE(9,*)'THE C MATRIX'
          WRITE(9,*)((C(I,J),J=1,6),I=1,3)
C         EXTRACT CPRIME FROM C
          DO 470 IROW=1,3
            DO 470 ICOL=1,3
 470           CPRIME(IROW,ICOL)=C(IROW,ICOL+3)
```

```
            WRITE(9,*)'THE CPRIME MATRIX'
            WRITE(9,*)((CPRIME(I,J),J=1,3),I=1,3)
            WRITE(*,*)'READY TO INVERT B MATRIX'
C       INVERT B MATRIX INTO DELX TO OBTAIN CORRECTIONS
             CALL LEQT2F(CPRIME,1,3,3,DELX,WORK,IER)
C       PRINT RESULTS OF THIS ITERATION
             WRITE(*,*)'IER=',IER
             WRITE(9,*)'CORRECTIONS THIS ITERATION:'
             WRITE(9,*)'dLR',DELX(1)
             WRITE(9,*)'dLU',DELX(2)
             WRITE(9,*)'dLV',DELX(3)
             WRITE(*,*)'dLR',DELL(1),'dLU',DELL(2),'dLV',DELL(3)
C       ADD CORRECTION TO TOTAL
            DO 600 I=1,3
 600          DTOT(I)=DTOT(I)-DELX(I)
            CLOSE(8)
            DO 700 I=1,3
 700        IF(DABS(DELL(I)).GT.1.0D-6) GOTO 2000
C           WRITE(*,*)'CONVERGEDDD'
            RF=X(1,NXT)
            WRITE(*,*)'RF=',RF,'RFOLD=',RFOLD,' RM=',RM
            WRITE(*,*)'TF=',TF,'TFOLD=',TFOLD
     .      TFNEW=TF+(TF-TFOLD)/(RF-RFOLD)*(RM-RF)
            DELTF=TFNEW-TF
            DELRM=RM-RF
            WRITE(*,*)'TFNEW=',TFNEW
            TFOLD=TF
            RFOLD=RF
            TF=TFNEW
            WRITE(9,*)'****************************************** '
C       CHECK FOR CONVERGENCE
C         WRITE(*,*)'CHECKING FOR CONVERGENCE'
              IF(DABS(DELRM).GT.1.0D-4) GOTO 2000
C       IT HAS CONVERGED
            GOTO 5000
 2000   CONTINUE
C       IT DID NOT WORK
         WRITE(*,*)'NO CONVERGENCE'
              STOP
C        SUCCESS
 5000    WRITE(*,*)'IT CONVERGED!'
 9999    WRITE(*,*)'PROGRAM COMPLETE'
         END

C
        SUBROUTINE RHS(NXT)
C        THIS CALCULATES THE EQUATIONS OF MOTION AND THE
C        EQUATIONS OF VARIATION
C        STATE VECTOR SPLIT OUT AS
C           X(1-3,NXT); RADIAL, RADIAL VELOCITY, TANGENTIAL VELOCITY VECT
ORS
C           X(4-6,NXT); RADIAL, RADIAL VEL, TANGENTIAL VEL LAGRANGE MULT'
S
C           X(7-42,NXT); IS THE STATE TRANSISTION MATRIX
```

```
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /HAM/ T,X(42,4),F(42,4),ERREST(42),N,H,MODE
      DOUBLE PRECISION A(6,6)
      XMU=1.0D00
      TH=.1405D0
      XMO=1.0D0
      XMDOT=.07487D0
C
C     THE BASIC FUNCTION OF RHS IS TO CALCULATE THE E.O.M.
C     TRANSLATE Y'S INTO SYMBOLS FOR EASY PROGRAMMING
      R=X(1,NXT)
      U=X(2,NXT)
      V=X(3,NXT)
      RL=X(4,NXT)
      UL=X(5,NXT)
      VL=X(6,NXT)
C     EVALUATE THRUST ACCELERATION TERM AND OTHER HELFUL TERMS
      TA=TH/(XMO-XMDOT*T)
      SQ=DSQRT(UL*UL+VL*VL)

C     EVALUATE EOM
      F(1,NXT)=U
      F(2,NXT)=V*V/R-XMU/(R*R)+TA*UL/SQ
      F(3,NXT)=-U*V/R+TA*VL/SQ
      F(4,NXT)=UL*(V*V/(R*R)-2.D0*XMU/(R*R*R))-VL*U*V/(R*R)
      F(5,NXT)=-RL+VL*V/R
      F(6,NXT)=-UL*2.D0*V/R+VL*U/R
C     EVALUATE COMMON A MATRIX FACTOR Q
      Q=TA/(UL*UL+VL*VL)**(1.5D0)
C     EQUATIONS OF VARIATION
C     FIRST CALCULATE THE A MATRIX
      A(1,1)=0.00D+00
      A(1,2)=1.00D+00
      A(1,3)=0.00D+00
      A(1,4)=0.00D+00
      A(1,5)=0.00D+00
      A(1,6)=0.00D+00
      A(2,1)=-V*V/(R*R)+2.D0*XMU/(R*R*R)
      A(2,2)=0.00D+00
      A(2,3)=2.D0*V/R
      A(2,4)=0.00D+00
      A(2,5)=-Q*UL*UL+TA/SQ
      A(2,6)=-Q*UL*VL
      A(3,1)=U*V/(R*R)
      A(3,2)=-V/R
      A(3,3)=-U/R
      A(3,4)=0.00D+00
      A(3,5)=-Q*UL*VL
      A(3,6)=-Q*VL*VL+TA/SQ
      A(4,1)=UL*(-2.D0*V*V/(R*R*R)+6.D0*XMU/R**4.D0)
     1   +VL*2.D0*U*V/(R*R*R)
      A(4,2)=-VL*V/(R*R)
```

```
      A(4,3)=2.D0*UL*V/(R*R)-VL*U/(R*R)
      A(4,4)=0.00D+00
      A(4,5)=V*V/(R*R)-2.D0*XMU/(R*R*R)
      A(4,6)=-U*V/(R*R)
      A(5,1)=-VL*V/(R*R)
      A(5,2)=0.00D+00
      A(5,3)=VL/R
      A(5,4)=-1.00D+00
      A(5,5)=0.00D+00
      A(5,6)=V/R
      A(6,1)=2.D0*UL*V/(R*R)-VL*U/(R*R)
      A(6,2)=VL/R
      A(6,3)=-2.D0*UL/R
      A(6,4)=0.00D+0
      A(6,5)=-2.D0*V/R
      A(6,6)=U/R
C     A MATRIX NOW CALCULATED
C     NOW CALCULATE PHIDOT=A*PHI  AND PUT INTO LAST 32 SLOTS
C     OF F MATRIX
C       DO LOOPS FOR ELEMENT OF PHI DOT
C       ROW NUMBER LOOP
      DO 800 II=1,6
C       COLUMN NUMBER LOOP
      DO 800 JJ=1,6
C       INITIALIZE ELEMENT
        IPOS=6*JJ+II
        F(IPOS,NXT)=0.00D+00
C       LOOP FOR MATRIX PRODUCT  A*PHI
      DO 700 KK=1,6
           JPOS =6*JJ+KK
C          MATRIX PRODUCT
C          write(*,*)'F(IPOS,NXT)',IPOS,NXT,F(IPOS,NXT)
C          WRITE(*,*)'A(II,KK)',II,KK,A(II,KK)
C          WRITE(*,*)'X(JPOS,NXT)',JPOS,NXT,X(JPOS,NXT)
           F(IPOS,NXT)=F(IPOS,NXT)+A(II,KK)*X(JPOS,NXT)
 700    CONTINUE
 800    CONTINUE
C      FINISHED
      RETURN
      END
C
C
```

```
      subroutine haming(nxt)
c
c     haming is an ordinary differential equations integrator
c     it is a fourth order predictor-corrector algorithm
c     which means that it carries along the last four
c     values of the state vector, and extrapolates these
c     values to obtain the next value (the prediction part)
c     and then corrects the extrapolated value to find a
c     new value for the state vector.
c
c     the value nxt in the call specifies which of the 4 values
c     of the state vector is the "next" one.
c     nxt is updated by haming automatically, and is zero on
c     the first call
c
c     the user supplies an external routine rhs(nxt) which
c     evaluates the equations of motion
c
      common /ham/ t,x(42,4),f(42,4),errest(42),n,h,mode
      double precision t,x,f,errest,h,hh,xo
c
c     all of the good stuff is in this common block.
c     t is the independent variable ( time )
c     x(6,4) is the state vector- 4 copies of it, with nxt
c                 pointing at the next one
c     f(6,4) are the equations of motion, again four copies
c                 a call to rhs(nxt) updates an entry in f
c     errest is an estimate of the truncation error - normally not
c                 used
c     n is the number of equations being integrated - 6 or 42 here
c     h is the time step
c     mode is 0 for just EOM, 1 for both EOM and EOV
c
      tol = 0.0000000001
c     switch on starting algorithm or normal propagation
      if(nxt) 190,10,200
c
c     this is hamings starting algorithm....a predictor - corrector
c     needs 4 values of the state vector, and you only have one- the
c     initial conditions.
c     haming uses a Picard iteration (slow and painfull) to get the
c     other three.
c     if it fails, nxt will still be zero upon exit, otherwise
c     nxt will be 1, and you are all set to go
c
   10 xo = t
      hh = h/2.0d+00
      call rhs(1)
      do 40 l = 2,4
      t = t + hh
      do 20 i = 1,n
   20 x(i,l) = x(i,l-1) + hh*f(i,l-1)
      call rhs(l)
      t = t + hh
```

```
      do 30 i = 1,n
   30 x(i,l) = x(i,l-1) + h*f(i,l)
   40 call rhs(l)
      jsw = -10
   50 isw = 1
      do 120 i = 1,n
      hh = x(i,1) + h*( 9.0d+00*f(i,1) + 19.0d+00*f(i,2)
     1     - 5.0d+00*f(i,3) + f(i,4) ) / 24.0d+00
      if( dabs( hh - x(i,2)) .lt. tol ) go to 70
      isw = 0
   70 x(i,2) = hh
      hh = x(i,1) + h*( f(i,1) + 4.0d+00*f(i,2) + f(i,3))/3.0d+00
      if( dabs( hh-x(i,3)) .lt. tol ) go to 90
      isw = 0
   90 x(i,3) = hh
      hh = x(i,1) + h*( 3.0d+00*f(i,1) + 9.0d+00*f(i,2) + 9.0d+00*f(i,3
)

     1     + 3.0d+00*f(i,4) ) / 8.0d+00
      if( dabs(hh-x(i,4)) .lt. tol ) go to 110
      isw = 0
  110 x(i,4) = hh
  120 continue
      t = xo
      do 130 l = 2,4
      t = t + h
  130 call rhs(l)
      if(isw) 140,140,150
  140 jsw = jsw + 1
      if(jsw) 50,280,280
  150 t = xo
      isw = 1
      jsw = 1
      do 160 i = 1,n
  160 errest(i) = 0.0
      nxt = 1
      go to 280
  190 jsw = 2
      nxt = iabs(nxt)
c
c     this is hamings normal propagation loop -
c
  200 t = t + h
      np1 = mod(nxt,4) + 1
      go to (210,230),isw
c     permute the index nxt modulo 4
  210 go to (270,270,270,220),nxt
  220 isw = 2
  230 nm2 = mod(np1,4) + 1
      nm1 = mod(nm2,4) + 1
      npo = mod(nm1,4) + 1
```

```
c
c     this is the predictor part
c
      do 240 i = 1,n
      f(i,nm2) = x(i,np1) + 4.0d+00*h*( 2.0d+00*f(i,npo) - f(i,nm1)
     1     + 2.0d+00*f(i,nm2) ) / 3.0d+00
  240 x(i,np1) = f(i,nm2) - 0.925619835*errest(i)
c
c     now the corrector - fix up the extrapolated state
c     based on the better value of the equations of motion
c
      call rhs(np1)
      do 250 i = 1,n
       x(i,np1) = ( 9.0d+00*x(i,npo) - x(i,nm2) + 3.0d+00*h*( f(i,np1)
     1     + 2.0d+00*f(i,npo) - f(i,nm1) ) ) / 8.0d+00
      errest(i) = f(i,nm2) - x(i,np1)
  250 x(i,np1) = x(i,np1) + 0.0743801653 * errest(i)
      go to (260,270),jsw
  260 call rhs(np1)
  270 nxt = np1
  280 return
      end
```

```
      subroutine leqt2f(a,m,n,nn,b,x,ier)
c
c     gaussian elimination with maximal pivoting
c     interface simulates IMSL routine
c     solution of a system of linear equations for m right sides
c     a: matrix of system
c     m: number of rhs
c     n: order of a, rows in b
c     ia: row dimension of a,b
c     b: right hand sides....solution on return
c     idgt: ignored here....in imsl 0=no acc test on input
c           idgt= #digits ok on output in imsl
c     x: in imsl, n**2 + 3*n
c     ier: 129: singular matrix, 0=ok
c
      dimension a(nn,nn),b(nn,m),irr(50),x(1)
      double precision a,b,x,anorm,amax,p,tol
c
c     find max norm of a
      anorm = 0.d0
      do 5 i = 1,n
           do 5 j = 1,n
                if(dabs(a(i,j)) .gt. anorm) anorm = dabs(a(i,j))
    5 continue
c     set tolerance = 2** (- number of binary digits in mantissa)
      tol = 1.d-12
      ier = 0
      id = 1
      do 10 i = 1,n
          irr(i) = 0
   10 continue
   20 ir = 1
      is = 1
      amax = 0.d0
c     find max pivot
      do 60 i = 1,n
          if(irr(i)) 60,30,60
   30     do 50 j = 1,n
              p = dabs(a(i,j))
              if(p-amax) 50,50,40
   40         ir = i
              is = j
              amax = p
   50     continue
   60 continue
c     singularity test
      if(amax/anorm .gt. tol) go to 70
      ier = 129
      go to 120
c     forward elimination
   70 irr(ir) = is
      do 90 i = 1,n
           if(i .eq. ir) go to 90
           p = a(i,is)/a(ir,is)
```

```
            do 80 j = 1,n
                 a(i,j) = a(i,j) - p*a(ir,j)
   80       continue
            a(i,is) = 0.0
            do 85 j = 1,m
                 b(i,j) = b(i,j) - p*b(ir,j)
   85       continue
   90  continue
       id = id + 1
       if(id .le. n) go to 20
c      back substitution
       do 115 j = 1,m
            do 100 i = 1,n
                 ir = irr(i)
                 x(ir) = b(i,j)/a(i,ir)
  100       continue
            do 110 i = 1,n
                 b(i,j) = x(i)
  110       continue
  115  continue
  120  return
       end
```

```
      PROGRAM OPTVEHICLE
C     PERFORMS INTEGRATION OF ORBIT AND DETERMINES PHI(T)
C     THRUST ANGLING FUNCTION SUCH THAT FINAL CONDITIONS
C     ARE SATISFIED AND TRANSFER TIME,
C     EXCESS FUEL DIVISION AND INJECTION ANGLE ARE OPTIMIZED
C     AS WELL AS ION DRIVE MASS VARIED
C     PROGRAMMER: STEVE OLESON
C     DATE: MAY 20, 1990
C     PROGRAM ALSO CREATES DATA FOR A 3-D PLOT
C     INPUTS HAVE BEEN NORMALIZED TO MASS RATIOS
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON X(42,4),F(42,4),T,H,VE,RMDOT,XMO,N,MODE
      DOUBLE PRECISION XIC(6),B(3,6),DELX(3),DTOT(3),CPRIME(3,3)
      DOUBLE PRECISION DELL(3),PHIMAT(6,6),C(3,6)
      EQUIVALENCE(DELX,DELL)
      PI=3.141592654
      TFMINI=100.0D0
C      FACTOR FOR CONVERTING SEC/TU'S AND KM/AU'S
      SECTU=(60.D0*60.D0*24.D0*58.132821D00)
      WRITE(*,*)SECTU
      XKMPAU=1.4959965D8
C      INITIAL LEO PARKING ALT
      RP=500.D0
      RP=RP+6378.145D0
C      MARS SOLAR POS. AND PARK ORBIT
      RM=1.523691D0
      RPARK=4380.D0/XKMPAU
C     POWER UNIT SPECIFIC POWER
      SPCPWM=.05D0
      SPCPWR=SPCPWM/1.D3*SECTU**3.0D0/(XKMPAU*XKMPAU)
      WRITE(*,*)'SPECIFIC PWR (AU*2/TU*3)',SPCPWR
C      NUCLEAR ENGINE PARAMETERS
      VH= 8.25D0
      BETA = 2.84D0
      ROCRAT = 1.D0/(2.D0*BETA)
      WRITE(*,*)'REQUIRED NUCLEAR ROCKET MASS RATIO:',ROCRAT
C      READ IN S/C MASS RATIOS
      WRITE(*,*)'INPUT PAYLOAD MASS RATIO'
      READ(*,*)PAYRAT
      WRITE(*,*)'INPUT DESIRED POWER GENERATION MASS RATIO'
      READ(*,*)PRWRAT
C      GUESS INITIAL LAGRANGE MULTIPLIERS
      WRITE(*,*)'INPUT LAMBDA R GUESS'
      READ(*,*)XIC(4)
      WRITE(*,*)'INPUT LAMBDA U GUESS'
      READ(*,*)XIC(5)
      WRITE(*,*)'INPUT LAMBDA V GUESS'
      READ(*,*)XIC(6)
      WRITE(*,*)'INPUT FINAL TIME GUESS'
       READ(*,*) TF
C      FIND ION PROPULSION MASSRATIO
       ELCRAT=2.0D0*PRWRAT
       WRITE(*,*)'ION PROP MASS RATIO=',ELCRAT
C      DETERMINE P1MIN MASS RATIO AND P1MAX MASS RATIO
```

```
      RP1MIN=(1.0D0-DEXP(-3.2D0/VH))
      RP1MAX=(1.0D0-ELCRAT-(PAYRAT+ROCRAT+PRWRAT)
     *   *DEXP(1.3/VH))
      OPEN(7,FILE = 'NRM2FIN')
      WRITE(7,*)'MASS RATIOS:'
      WRITE(7,17)PAYRAT
17    FORMAT('PAYLOAD:                ',1F5.4)
      WRITE(7,11)ROCRAT
11    FORMAT('NUCLEAR PROPULSION:',1F5.4)
      WRITE(7,12)PRWRAT
12    FORMAT('POWER GENERATION:  ',1F5.4)
      WRITE(7,13)ELCRAT
13    FORMAT('ION PROPELLANT:    ',1F5.4)
      WRITE(7,14)RP1MIN,RP1MAX
14    FORMAT('RP1MIN= ',1F5.4,'  RP1MAX= ',1F5.4)
      WRITE(7,15)BETA
15    FORMAT('BETA (THRUST/WT RATIO)=',1F8.4)
      WRITE(7,16)SPCPWM
16    FORMAT('SP. POWER ALPHA (M*M/S*S*S)=',1F10.5)
      WRITE(*,*)'RP1MIN=',RP1MIN,'RP1MAX=',RP1MAX
      IF(RP1MIN.GT.RP1MAX) STOP
10    FORMAT(1F5.4)
      WRITE(*,*)'INPUT MIN AND MAX P1RAT EXCESS PERCENTAGE'
      READ(*,*)PCTMIN
      READ(*,*)PCTMAX
      PEX=RP1MAX-RP1MIN
      WRITE(*,*)'EXCESS FUEL RATIO = ',PEX
      TP1MAX=PCTMAX/1.D2*PEX + RP1MIN
      TP1MIN=PCTMIN/1.D2*PEX + RP1MIN
55    WRITE(*,*)'INPUT EXCESS PERCENTATGE STEP SIZE'
      READ(*,*)PCTSTP
      P1STEP=PCTSTP/1.D2*PEX
      WRITE(*,*)'INPUT LOW AND HIGH PSI VALUES (DEG)'
      READ(*,*)PSILOW,PSIHI
      WRITE(*,*)'INPUT PSI STEP'
      READ(*,*)PSISTP
      IF(PSISTP.LT.0.D0) GOTO 9999
      OPEN(2,FILE='NRM2DAT')
C     OUTER LOOP TO RUN FROM P1MIN TO P1MAX
      P1RAT=TP1MIN
66    WRITE(*,*)'P1RATIO=',P1RAT
C     DETERMINE DELTAV1,DELTAV2
      DELV1=VH*DLOG(1/(1-P1RAT))
      DELTA2=VH*DLOG((1-ELCRAT-P1RAT)/(PAYRAT+ROCRAT+PRWRAT))
      WRITE(*,*)'DELTA V1=',DELV1
      WRITE(*,*)'DELTA V2=',DELTA2
      DELV2=DELTA2/29.784852D0
C
C     CALCULATE INITIAL STATE
      XMUE=3.986D5
      ANGMOM=RP*(DELV1+DSQRT(XMUE/RP))
      ENERGY=.5D0*(DELV1+DSQRT(XMUE/RP))**2.0D0-XMUE/RP
```

```
      XNU=DACOS(-(1.D0+2.D0*ENERGY*ANGMOM**2.D0/(XMUE*XMUE))**(-.5D0))
      VINF=DSQRT((DELV1+DSQRT(XMUE/RP))**2.D0-2.D0*XMUE/RP)
C     CONVERT TO HELIOCENTRIC UNITS
      VINF=VINF/29.784852D0
C     DEFINE PSI SEARCH RANGE
C     PSILOW=-30.D0
C     PSIHI=30.D0
C     ********************************
C      LOOP FOR PSI SEARCH RANGE
      DO 3000 PSIDEG=PSILOW,PSIHI,PSISTP
      RADEG=0.017453293D0
      PSI=PSIDEG*RADEG
C      FIND INITIAL STATE VECTOR
       XIC(1)=1.0D0
       XIC(2)=VINF*DCOS(XNU+PSI-PI/2.0D0)
       XIC(3)=1.0D0+VINF*DSIN(XNU+PSI-PI/2.0D0)
C
      TFOLD=TF-TF/50.D0
      RFOLD=RM-RM/50.D0
C
C     MAX ITERATIONS, # OF STEPS
      MAXIT= 50
      NSTEP=1000
C
C          SET UP HAMING
      N=42
      T=0.0D0
      MODE=1
C
C     LOOP ON ITERATIONS
      DO 190 II=1,3
         DTOT(II)=0.00D+00
         DELX(II)=0.00D+00
 190     DELL(II)=0.00D+00
C
C     OPEN (9,FILE = 'THREE.OUT')
C     MAIN PROGRAM LOOP
      DO 2000 ITER=1,25
       NSTEP=1000
       DSTEP=NSTEP
       H=TF/DSTEP
C      CALCULATE MDOT RATIO
       VE=DSQRT(SPCPWR*TF)
       RMDOT=2.D0*SPCPWR*PRWRAT/(VE*VE*(1.0D0-P1RAT))
       OPEN (8,FILE = 'ANGLE')
C      INITIALIZE ERROR VECTORS
        DO 195 J=1,3
          DELX(J)=0.00D+00
 195      DELL(J)=0.00D+00
C
C       SET UP IC'S FOR ORBIT, PHI INTEGRATION
C       INITIAL X(1-3,NXT) NEVER CHANGES, CHANGE LAGRANGE FACTORS
          DO 200 II=1,3
 200          X(II,1)=XIC(II)
```

```
        DO 205 II=1,3
             X(II+3,1)=XIC(II+3)+DTOT(II)
 205      CONTINUE
C        INTIALIZE PHI MATRIX
          DO 210 II=7,42
 210         X(II,1)=0.00D0
          DO 220 II=7,42,7
 220         X(II,1)=1.00D+00
C
C         RESET INITIAL TIME
               T=0.00D0
C
C         INTEGRATE ORBIT
          NXT=0
C          WRITE(*,*)'GO TO HAMING 1'
          MODE=1
          CALL HAMING(NXT)
           IF(NXT.EQ.0) THEN
               WRITE(*,*) 'HAMING FAILED!'
               GO TO 3000
           ENDIF
C
C          MAIN HAMING INTEGRATION LOOP
           JJJ=10
           DO 300 I=1,1000
              MODE=1
              CALL HAMING(NXT)
               FPHI=DATAN2(X(5,NXT),X(6,NXT))
                JJJ = JJJ + 1
                IF (JJJ.GE.10) THEN
                     FPHI=FPHI*57.29577951D0
                     IF(FPHI.LT.0) FPHI=FPHI+360.0D0
                     WRITE(8,299)T*58.132821D00,FPHI,X(1,NXT)
 299                 FORMAT(3F12.6)
                     JJJ=0
                END IF
 300       CONTINUE
           CLOSE(8)
C
C      ORBIT INTEGRATED
C         SIMPLIFY VARIABLES
           R=X(1,NXT)
           U=X(2,NXT)
           V=X(3,NXT)
           RL=X(4,NXT)
           UL=X(5,NXT)
           VL=X(6,NXT)

C         FIND HELPFUL FACTORS
           XMUM=4.305D4/1.3271544D11
           XMU=1.0D0
           VDIFF=V-DSQRT(XMU/R)
           PE=XMUM/RPARK
           VCDOT=.50D0*DSQRT(XMU)/(R**1.5D0)
C
```

```
C     CALCULATE ERROR VECTOR
           XMU=1.0D0
       DELX(1)=U*U+VDIFF*VDIFF+2.0D0*PE-(DELV2+DSQRT(PE))**2.0D0
       DELX(2)=RL-VL*VCDOT-1.0D0
       DELX(3)=UL*VDIFF-VL*U
           DO 33 I=1,3
 33        IF(DELX(I).GT.1.D3) GOTO34
C          TRANSFORM PHI TO NORMAL 6X6
           ICOUNT=0
           DO 400 ICOL=1,6
              ICOUNT=ICOUNT + 6
             DO 400 IROW=1,6
              PHIMAT(IROW,ICOL)=X(ICOUNT+IROW,NXT)
 400        CONTINUE
C         CALCULATE THE B MATRIX
           DO 410 I=1,3
            DO 410 J=1,6
 410            B(I,J)=0.D0
           B(1,1)=2.0D0*VDIFF*VCDOT
           B(1,2)=2.0D0*U
           B(1,3)=2.0D0*VDIFF
           B(2,1)=.75D0*VL*DSQRT(XMU)/(R**2.5D0)
           B(2,4)=1.0D0
           B(2,6)=-VCDOT
           B(3,1)=UL*VCDOT
           B(3,2)=-VL
           B(3,3)=UL
           B(3,5)=VDIFF
           B(3,6)=-U
C
C          MULIPITPLY C=B*PHI
           DO 460 II=1,3
            DO 460 JJ=1,6
              C(II,JJ)=0.0D0
              DO 450 KK=1,6
               C(II,JJ)=C(II,JJ)+B(II,KK)*PHIMAT(KK,JJ)
 450          CONTINUE
 460       CONTINUE
C
C          EXTRACT CPRIME FROM C
          DO 470 IROW=1,3
            DO 470 ICOL=1,3
 470           CPRIME(IROW,ICOL)=C(IROW,ICOL+3)
C
C     INVERT B MATRIX INTO DELX TO OBTAIN CORRECTIONS
           CALL LEQT2F(CPRIME,1,3,3,DELX,WORK,IER)
C     PRINT RESULTS OF THIS ITERATION
C          WRITE(*,*)'IER=',IER
           IF(IER.NE.129) GO TO 490
              WRITE(*,*)'SINGULAR MATRIX'
             GOTO 3000
 490        CONTINUE
C     ADD CORRECTION TO TOTAL
```

```
          DO 600 I=1,3
 600        DTOT(I)=DTOT(I)-DELX(I)
          CLOSE(8)
          DO 700 I=1,3
 700      IF(DABS(DELL(I)).GT.1.0D-6) GOTO 2000
          RF=X(1,NXT)
          TFNEW=TF+(TF-TFOLD)/(RF-RFOLD)*(RM-RF)
          DELTF=TFNEW-TF
          DELRM=RM-RF
C         WRITE(*,*)'TFNEW=',TFNEW
          TFOLD=TF
          RFOLD=RF
          TF=TFNEW
C     CHECK FOR CONVERGENCE
            IF(DABS(DELRM).GT.1.0D-6) GOTO 2000
C     IT HAS CONVERGED
          GOTO 5000
 2000 CONTINUE
C     IT DID NOT WORK
 34    WRITE(*,*)'NO CONVERGENCE'
            GOTO 3000
C      SUCCESS
 5000  CONTINUE
       DO 5010 I=1,3
 5010        XIC(I+3)=XIC(I+3)+DTOT(I)
        PEX=RP1MAX-RP1MIN
       WRITE(2,*)1.D2*(P1RAT-RP1MIN)/PEX,PSIDEG,TFOLD*58.132821D00
       WRITE(*,*)1.D2*(P1RAT-RP1MIN)/PEX,PSIDEG,TFOLD*58.132821D00
        TFTEST=TFOLD
       IF(TFTEST.LT.TFMINI) THEN
        TFMINI=TFTEST
        PCTMIN=1.D2*(P1RAT-RP1MIN)/PEX
        PSIMIN=PSIDEG
       END IF
       WRITE(*,*)'OVERALL MINTIME (DAYS)',TFMINI*58.132821D00
 3000   CONTINUE
       P1RAT=P1RAT+P1STEP
       IF(P1RAT.LE.TP1MAX) GOTO 66
       WRITE(7,5005)VE*29.78485D0
 5005  FORMAT('VE:OPTIMUM (KM/S)=',F7.4)
       WRITE(7,3001)TFMINI*58.132821D00
 3001  FORMAT('OVERALL MINTIME (DAYS)=',F7.3)
       WRITE(7,3002)PCTMIN
 3002  FORMAT('OPTIMAL PERCENT OF EXCESS TO DV1=',F5.2)
       WRITE(7,3003)PEX
 3003  FORMAT('EXCESS FUEL RATIO= ',F5.4)
       WRITE(7,3004)PSIMIN
 3004  FORMAT('OPTIMAL INJECTION ANGLE (DEG)=',F6.2)
       WRITE(7,*)'THE PARAMETERS OF THE MINIMUM RUN'
       WRITE(7,*)'THE CONVERGED INITIAL CONDITIONS'
       WRITE(7,3005)(XIC(II),II=1,6)
 3005  FORMAT(6F10.6)
 9999  WRITE(*,*)'PROGRAM COMPLETE'
       END
```

```
      SUBROUTINE RHS(NXT)
C      THIS CALCULATES THE EQUATIONS OF MOTION AND THE
C      EQUATIONS OF VARIATION
C      STATE VECTOR SPLIT OUT AS
C         X(1-3,NXT); RADIAL, RADIAL VELOCITY, TANGENTIAL VELOCITY VECT
)
C         X(4-6,NXT); RADIAL, RADIAL VEL, TANGENTIAL VEL LAGRANGE MULT'
S
C         X(7-42,NXT); IS THE STATE TRANSISTION MATRIX
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON X(42,4),F(42,4),T,H,VE,RMDOT,XMO,N,MODE
      DOUBLE PRECISION A(6,6)
      XMU=1.0D00
C
C     THE BASIC FUNCTION OF RHS IS TO CALCULATE THE E.O.M.
C     TRANSLATE Y'S INTO SYMBOLS FOR EASY PROGRAMMING
      R=X(1,NXT)
      U=X(2,NXT)
      V=X(3,NXT)
      RL=X(4,NXT)
      UL=X(5,NXT)
      VL=X(6,NXT)
C     EVALUATE THRUST ACCELERATION TERM AND OTHER HELFUL TERMS
      TA=RMDOT*VE/(1.0D0-RMDOT*T)
      SQ=DSQRT(UL*UL+VL*VL)
C
C     EVALUATE EOM
      F(1,NXT)=U
      F(2,NXT)=V*V/R-XMU/(R*R)+TA*UL/SQ
      F(3,NXT)=-U*V/R+TA*VL/SQ
      F(4,NXT)=UL*(V*V/(R*R)-2.D0*XMU/(R*R*R))-VL*U*V/(R*R)
      F(5,NXT)=-RL+VL*V/R
      F(6,NXT)=-UL*2.D0*V/R+VL*U/R
C     EVALUATE COMMON A MATRIX FACTOR Q
      Q=TA/(UL*UL+VL*VL)**(1.5D0)
C     EQUATIONS OF VARIATION
C     FIRST CALCULATE THE A MATRIX
      A(1,1)=0.00D+00
      A(1,2)=1.00D+00
      A(1,3)=0.00D+00
      A(1,4)=0.00D+00
      A(1,5)=0.00D+00
      A(1,6)=0.00D+00
      A(2,1)=-V*V/(R*R)+2.D0*XMU/(R*R*R)
      A(2,2)=0.00D+00
      A(2,3)=2.D0*V/R
      A(2,4)=0.00D+00
      A(2,5)=-Q*UL*UL+TA/SQ
      A(2,6)=-Q*UL*VL
      A(3,1)=U*V/(R*R)
      A(3,2)=-V/R
      A(3,3)=-U/R
      A(3,4)=0.00D+00
      A(3,5)=-Q*UL*VL
      A(3,6)=-Q*VL*VL+TA/SQ
```

```
      A(4,1)=UL*(-2.D0*V*V/(R*R*R)+6.D0*XMU/R**4.D0)
     1    +VL*2.D0*U*V/(R*R*R)
      A(4,2)=-VL*V/(R*R)
      A(4,3)=2.D0*UL*V/(R*R)-VL*U/(R*R)
      A(4,4)=0.00D+00
      A(4,5)=V*V/(R*R)-2.D0*XMU/(R*R*R)
      A(4,6)=-U*V/(R*R)
      A(5,1)=-VL*V/(R*R)
      A(5,2)=0.00D+00
      A(5,3)=VL/R
      A(5,4)=-1.00D+00
      A(5,5)=0.00D+00
      A(5,6)=V/R
      A(6,1)=2.D0*UL*V/(R*R)-VL*U/(R*R)
      A(6,2)=VL/R
      A(6,3)=-2.D0*UL/R
      A(6,4)=0.00D+0
      A(6,5)=-2.D0*V/R
      A(6,6)=U/R
C     A MATRIX NOW CALCULATED
C     NOW CALCULATE PHIDOT=A*PHI  AND PUT INTO LAST 32 SLOTS
C     OF F MATRIX
C       DO LOOPS FOR ELEMENT OF PHI DOT
C       ROW NUMBER LOOP
      DO 800 II=1,6
C       COLUMN NUMBER LOOP
      DO 800 JJ=1,6
C       INITIALIZE ELEMENT
        IPOS=6*JJ+II
        F(IPOS,NXT)=0.00D+00
C       LOOP FOR MATRIX PRODUCT  A*PHI
      DO 700 KK=1,6
           JPOS =6*JJ+KK
           F(IPOS,NXT)=F(IPOS,NXT)+A(II,KK)*X(JPOS,NXT)
 700    CONTINUE
 800    CONTINUE
C      FINISHED
      RETURN
      END
```

```
      PROGRAM IMPULSE
C     PERFORMS INTEGRATION OF ESCAPE ORBIT FOR
C     A HIGH THRUST ENGINE
C     PROGRAMMER: STEVE OLESON
C     DATE: MAY 20, 1990
C     INPUTS HAVE BEEN NORMALIZED TO MASS RATIOS
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON X(42,4),F(42,4),T,H,VE,RMDOT,TBURN,N,MODE
      DOUBLE PRECISION XIC(3)
      PI=3.141592654
       OPEN(7,FILE = 'FINAL')
C      FACTOR FOR CONVERTING SEC/TU'S AND KM/DU'S
      KMPDU=6378.145D0
      SECPTU=806.8118744D0
      XMUE=1.D0
       G0=9.81D0
C      EARTH'S SPHERE OF INFLUENCE (KM)
       ESOI=924118.5D0
       ESOI=ESOI/KMPDU
C      INITIAL PARKING ORBIT
       RP=500.D0
       RP=(RP+6378.145D0)
C      PROPULSION UNIT SPECIFIC POWER
       SPCPWR=.03D0
       SPCPWR=SPCPWR/1.D3*SECPTU**3.0D0*(KMPDU*KMPDU)
C      WRITE(*,*)'SPECIFIC PWR (AU*2/TU*3)',SPCPWR
C      NUCLEAR ENGINE PARAMETERS
C      READ IN S/C MASS RATIOS
      WRITE(*,*)'INPUT PAYLOAD MASS RATIO'
      READ(*,*)PAYRAT
      WRITE(7,*)'MASS RATIOS:'
      WRITE(7,*)'PAYLOAD=',PAYRAT
      WRITE(*,*)'INPUT PROPULSION UNIT MASS RATIO'
      READ(*,*)ROCRAT
      WRITE(7,*)'PROPULSION UNIT=',ROCRAT
      WRITE(*,*)'INPUT NUCLEAR DRIVE EXHAUST VELOCITY (KM/S)'
      READ(*,*)VHM
      WRITE(*,*)'INPUT INITIAL THRUST/WT RATIO BETA '
      READ(*,*)BETA
      WRITE(7,*)'THRUST/WT RATIO BETA ',BETA
      RMDOT=BETA*G0*ROCRAT/(VHM*1000.0D0)
      WRITE(*,*)'SP.MASS FLOW RATE(HIGH THRUST)(/SEC)=',RMDOT
      WRITE(*,*)'INPUT FINAL TIME GUESS (DAYS)'
       READ(*,*) TF
C     TRANSFORM TO CANONICAL UNITS
      VH=VHM*SECPTU/KMPDU
      RP=RP/KMPDU
      RMDOT=RMDOT*SECPTU
      TF=TF*24.D0*60.D0*60.D0/SECPTU
C      FIND ION PROPULSION MASSRATIO
       ELCRAT=2.0D0*ROCRAT
       WRITE(*,*)'ION PROP MASS RATIO=',ELCRAT
C      DETERMINE P1MIN MASS RATIO AND P1MAX MASS RATIO
```

```
      RP1MIN=(1.0D0-DEXP(-3.2D0/VHM))
      RP1MAX=(1.0D0-ELCRAT-(PAYRAT+ROCRAT)*DEXP(1.3D0/VHM))
      WRITE(*,*)'MASS RATIOS:'
      WRITE(*,*)'PAYLOAD:          ',PAYRAT
      WRITE(*,*)'PROPULSION:       ',ROCRAT
      WRITE(7,*)'ION PROPELLANT: ',ELCRAT
      WRITE(*,*)'RP1MIN=',RP1MIN,'RP1MAX=',RP1MAX
      IF(RP1MIN.GT.RP1MAX) STOP
      WRITE(7,*)'RP1MIN=',RP1MIN,'RP1MAX=',RP1MAX
      WRITE(7,*)'PROPELLANT FOR V1=',P1RAT
      WRITE(*,*)'INPUT P1 RATIO'
      READ(*,*)P1RAT
      WRITE(*,*)'TIME OF BURN OUT (MINS)=',TBURN*SECPTU/60.D0
C     WRITE(*,*)'INPUT P1RATIO STEP SIZE'
C     READ(*,*)P1STEP
      WRITE(7,*)'BURN TIME(MIN)=',TBURN*SECPTU/60.D0
      OPEN(2,FILE='IMPDAT')
C     ++++++++++++++++++++++++++++++++++++
C     OUTER LOOP TO RUN FROM P1MIN TO P1MAX
       INST=1
      BETSTP=1.0D0
      BETMAX=10.0D0
 66   WRITE(*,*)'BETA*ROCRAT=',BETA*ROCRAT
C     CALCULATE NEW SPECIFIC FLOW RATE
     RMDOT=BETA*G0*ROCRAT/(VHM*1000.0D0)*SECPTU
C    DETERMINE DELTAV1,DELTAV2
      TBURN=P1RAT/RMDOT
      DELV1=VHM*DLOG(1/(1-P1RAT))
      DELTA2=VHM*DLOG((1-ELCRAT-P1RAT)/(PAYRAT+ROCRAT))
      WRITE(7,*)'IMPULSIVE DELTA VEES (KM/SEC)'
      WRITE(7,*)'DELTA V1=',DELV1
      WRITE(7,*)'DELTA V2=',DELTA2
C     ASSUME INSTANTANEOUS OR ACTUAL BURN
C     WRITE(*,*)'INSTANTANEOUS BURN (1=YES)'
C     READ(*,*)INST
      WRITE(*,*)'DELV1 (KM/S)=',DELV1
      IF(INST.EQ.1) TBURN=0.D0
      IF(INST.NE.1) DELV1=0.D0
      IF(INST.EQ.1) WRITE(7,*)'INSTANTANEOUS'
C     FIND INITIAL STATE VECTOR
      XIC(1)=RP
      XIC(2)=0.0D0
      XIC(3)=DSQRT(XMUE/RP)+DELV1*SECPTU/KMPDU
C
C
C    MAX ITERATIONS, # OF STEPS
     MAXIT= 50
     NSTEP=1000000
C
C         SET UP HAMING
     N=3
     T=0.0D0
     MODE=0
C
```

```
C     MAIN PROGRAM LOOP
       NSTEP=1000000
       DSTEP=NSTEP
       H=TF/DSTEP
C      CALCULATE MDOT RATIO
       VE=VH
C
C       SET UP IC'S FOR ORBIT, PHI INTEGRATION
C       INITIAL X(1-3,NXT) NEVER CHANGES
          DO 200 II=1,3
 200          X(II,1)=XIC(II)
C
C
C        RESET INITIAL TIME
              T=0.00D0
C
C        INTEGRATE ORBIT
          NXT=0
           WRITE(*,*)'GO TO HAMING 1'
          MODE=0
          CALL HAMING(NXT)
           IF(NXT.EQ.0) THEN
               WRITE(*,*) 'HAMING FAILED!'
               GO TO 9999
           ENDIF
C
C         MAIN HAMING INTEGRATION LOOP
           JJJ=10000
           DO 300 I=1,1000000
              MODE=0
              CALL HAMING(NXT)
                JJJ = JJJ + 1
                IF(X(1,NXT).GE.ESOI) GOTO 301
                IF (JJJ.GE.10000) THEN
      WRITE(*,*)'T (DAYS)=',T*9.33D-3,'R(KM)=',X(1,NXT)*6378.145D0
C     WRITE(*,*)T*9.33D-3,X(1,NXT)*6378.145D0
                     JJJ=0
                END IF
 300       CONTINUE
C
C     ORBIT INTEGRATED
 301       WRITE(7,*)'THE FINAL STATE VECTOR'
           WRITE(7,*)(X(I,NXT),I=1,3)
      WRITE(*,*)'T (DAYS)=',T*9.33D-3,'R(KM)=',X(1,NXT)*6378.145D0
C      SUCCESS
       IF(INST.NE.1) GOTO 302
        TIMP=T
        INST=2
       GOTO 66
 302      WRITE(7,*)'FINAL RADIAL POSITION (KM)=',X(1,NXT)*KMPDU
       IF(X(1,NXT).LT.ESOI) WRITE(7,*)'FAILED TO ESCAPE!!!'
       WRITE(7,*)'FLIGHT TIME(DAYS)=',T*SECPTU/60.D0/60.D0/24.D0
       WRITE(2,*)T/TIMP,BETA*ROCRAT
```

```
      BETA=BETA+BETSTP
      IF(BETA.LT.BETMAX)GOTO 66
 9999 WRITE(*,*)'PROGRAM COMPLETE'
      END

      SUBROUTINE RHS(NXT)
C      THIS CALCULATES THE EQUATIONS OF MOTION AND THE
C      EQUATIONS OF VARIATION
C      STATE VECTOR SPLIT OUT AS
C         X(1-3,NXT); RADIAL, RADIAL VELOCITY, TANGENTIAL VELOCITY VECT

C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON X(42,4),F(42,4),T,H,VE,RMDOT,TBURN,N,MODE
      XMUE=1.0D00
C
C     THE BASIC FUNCTION OF RHS IS TO CALCULATE THE E.O.M.
C     TRANSLATE Y'S INTO SYMBOLS FOR EASY PROGRAMMING
      R=X(1,NXT)
      U=X(2,NXT)
      V=X(3,NXT)
C     EVALUATE THRUST ACCELERATION TERM AND OTHER HELFUL TERMS
      TA=RMDOT*VE/(1.0D0-RMDOT*T)
      IF(T.GT.TBURN) TA=0.0D0
C     EVALUATE EOM
      F(1,NXT)=U
      F(2,NXT)=V*V/R-XMUE/(R*R)
      F(3,NXT)=-U*V/R+TA
C      FINISHED
      RETURN
      END
```

APPENDIX C: DATA

DATA FOR SAMPLE VEHICLE OPTIMIZATIONS

```
 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0050
ION PROPELLANT:    .0100
RP1MIN= .3215  RP1MAX= .6610
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=22.3383
OVERALL MINTIME (DAYS)=115.279
OPTIMAL PERCENT OF EXCESS TO DV1=44.00
EXCESS FUEL RATIO= .3395
OPTIMAL INJECTION ANGLE (DEG)= -7.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.213033  1.116638  0.711100  0.557222 -0.192002


 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0100
ION PROPELLANT:    .0200
RP1MIN= .3215  RP1MAX= .6451
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=22.5237
OVERALL MINTIME (DAYS)=116.648
OPTIMAL PERCENT OF EXCESS TO DV1=45.00
EXCESS FUEL RATIO= .3236
OPTIMAL INJECTION ANGLE (DEG)= -7.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.204966  1.116817  0.463243  0.461015 -0.572078

 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0200
ION PROPELLANT:    .0400
RP1MIN= .3215  RP1MAX= .6134
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=22.7597
OVERALL MINTIME (DAYS)=119.513
OPTIMAL PERCENT OF EXCESS TO DV1=45.00
EXCESS FUEL RATIO= .2919
OPTIMAL INJECTION ANGLE (DEG)= -6.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.191909  1.116856  0.867513  0.637992  0.085610
```

```
 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0300
ION PROPELLANT:    .0600
RP1MIN= .3215  RP1MAX= .5817
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=23.0248
OVERALL MINTIME (DAYS)=122.608
OPTIMAL PERCENT OF EXCESS TO DV1=46.00
EXCESS FUEL RATIO= .2602
OPTIMAL INJECTION ANGLE (DEG)= -5.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.174908  1.116382  1.011687  0.702748  0.330558


 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0400
ION PROPELLANT:    .0800
RP1MIN= .3215  RP1MAX= .5500
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=23.3499
OVERALL MINTIME (DAYS)=125.988
OPTIMAL PERCENT OF EXCESS TO DV1=48.00
EXCESS FUEL RATIO= .2285
OPTIMAL INJECTION ANGLE (DEG)= -5.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.158739  1.121255  1.019420  0.701648  0.359720


 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0500
ION PROPELLANT:    .1000
RP1MIN= .3215  RP1MAX= .5183
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=23.6835
OVERALL MINTIME (DAYS)=129.736
OPTIMAL PERCENT OF EXCESS TO DV1=49.00
EXCESS FUEL RATIO= .1968
OPTIMAL INJECTION ANGLE (DEG)= -4.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.147064  1.116908  1.085455  0.726499  0.474641
```

```
 MASS RATIOS:
PAYLOAD:               .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0600
ION PROPELLANT:    .1200
RP1MIN= .3215  RP1MAX= .4866
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=24.0631
OVERALL MINTIME (DAYS)=133.994
OPTIMAL PERCENT OF EXCESS TO DV1=51.00
EXCESS FUEL RATIO= .1651
OPTIMAL INJECTION ANGLE (DEG)= -4.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.127314  1.113663  1.148128  0.761763  0.599011

 MASS RATIOS:
PAYLOAD:               .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0700
ION PROPELLANT:    .1400
RP1MIN= .3215  RP1MAX= .4549
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=24.5100
OVERALL MINTIME (DAYS)=138.990
OPTIMAL PERCENT OF EXCESS TO DV1=53.00
EXCESS FUEL RATIO= .1334
OPTIMAL INJECTION ANGLE (DEG)= -5.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.113660  1.110607  1.178269  0.773602  0.662861

 MASS RATIOS:
PAYLOAD:               .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0800
ION PROPELLANT:    .1600
RP1MIN= .3215  RP1MAX= .4232
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=25.0541
OVERALL MINTIME (DAYS)=145.132
OPTIMAL PERCENT OF EXCESS TO DV1=55.00
EXCESS FUEL RATIO= .1017
OPTIMAL INJECTION ANGLE (DEG)= -5.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.089856  1.107842  1.223386  0.799135  0.771945
```

```
 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .0900
ION PROPELLANT:    .1800
RP1MIN= .3215  RP1MAX= .3915
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=25.7487
OVERALL MINTIME (DAYS)=153.242
OPTIMAL PERCENT OF EXCESS TO DV1=58.00
EXCESS FUEL RATIO= .0700
OPTIMAL INJECTION ANGLE (DEG)= -6.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.068350  1.098093  1.301691  0.834731  0.936159


 MASS RATIOS:
PAYLOAD:              .1000
NUCLEAR PROPULSION:.1761
POWER GENERATION:  .1000
ION PROPELLANT:    .2000
RP1MIN= .3215  RP1MAX= .3598
BETA (THRUST/WT RATIO)=  2.8400
SP. POWER ALPHA (M*M/S*S*S)=   0.05000
VE:OPTIMUM (KM/S)=26.7634
OVERALL MINTIME (DAYS)=165.397
OPTIMAL PERCENT OF EXCESS TO DV1=60.00
EXCESS FUEL RATIO= .0383
OPTIMAL INJECTION ANGLE (DEG)= -8.00
 THE PARAMETERS OF THE MINIMUM RUN
 THE CONVERGED INITIAL CONDITIONS
  1.000000  0.044118  1.082504  1.420957  0.876365  1.182654
```

DATA FOR ADVANCED TECHNOLOGY VEHICLE OPTIMIZATIONS

MASS RATIOS:
PAYLOAD: 0.100
NUCLEAR PROPULSION: 0.166
POWER GENERATION: 0.500E-02
ION PROPELLANT: 0.102E-01
RP1MIN= 0.299 RP1MAX= 0.677
BETA (THRUST/WT RATIO)= 3.00
SP. POWER ALPHA (M*M/S*S*S)= 0.1000
OVERALL MINTIME (DAYS)= 103.740
PERCENT OF EXCESS TO DV1= 45.00
EXCESS FUEL RATIO= 0.378
INJECTION ANGLE (DEG)= -5.000
THE PARAMETERS OF THE MINIMUM RUN
THE INITIAL CONDITIONS

1.00	0.2667	1.2888
1.530	0.7848	0.8809

MASS RATIOS:
PAYLOAD: 0.100
NUCLEAR PROPULSION: 0.166
POWER GENERATION: 0.250E-01
ION PROPELLANT: 0.500E-01
RP1MIN= 0.299 RP1MAX= 0.621
BETA (THRUST/WT RATIO)= 3.000
SP. POWER ALPHA (M*M/S*S*S)= 0.1000
OVERALL MINTIME (DAYS)= 105.802
PERCENT OF EXCESS TO DV1= 45.00
EXCESS FUEL RATIO= 0.322
INJECTION ANGLE (DEG)= 0.0000E+00
THE PARAMETERS OF THE MINIMUM RUN
THE INITIAL CONDITIONS

1.00	0.2702	1.290
0.843	0.2946	-0.461E-01

MASS RATIOS:
PAYLOAD: 0.1000
NUCLEAR PROPULSION: 0.1666
POWER GENERATION: 0.5000E-01
ION PROPELLANT: 0.1000E-01
RP1MIN= 0.299 RP1MAX= 0.551
BETA (THRUST/WT RATIO)= 3.000
SP. POWER ALPHA (M*M/S*S*S)= 0.1000
OVERALL MINTIME (DAYS)= 108.55
PERCENT OF EXCESS TO DV1= 55.00
EXCESS FUEL RATIO= 0.252
INJECTION ANGLE (DEG)= 0.000E+00
THE PARAMETERS OF THE MINIMUM RUN
THE INITIAL CONDITIONS

1.000	0.203	1.260
0.777	0.262	-0.141

```
MASS RATIOS:
PAYLOAD:                0.1000
NUCLEAR PROPULSION: 0.1666
POWER GENERATION:   0.7500E-01
ION PROPELLANT:     0.15
RP1MIN= 0.299     RP1MAX= 0.481
BETA (THRUST/WT RATIO)=  3.000
SP. POWER ALPHA (M*M/S*S*S)= 0.100
OVERALL MINTIME (DAYS)=  112.19
PERCENT OF EXCESS TO DV1=  55.00
EXCESS FUEL RATIO=  0.182
INJECTION ANGLE (DEG)= 0.000E+00
THE PARAMETERS OF THE MINIMUM RUN
THE INITIAL CONDITIONS
  1.000      0.138       1.227
  0.757      0.477      -0.822

MASS RATIOS:
PAYLOAD:                0.100
NUCLEAR PROPULSION: 0.166
POWER GENERATION:   0.100
ION PROPELLANT:     0.200
RP1MIN= 0.299     RP1MAX= 0.411
BETA (THRUST/WT RATIO)=  3.000
SP. POWER ALPHA (M*M/S*S*S)= 0.1000
OVERALL MINTIME (DAYS)=  117.66
PERCENT OF EXCESS TO DV1=  65.00
EXCESS FUEL RATIO=  0.111
INJECTION ANGLE (DEG)= -5.000
THE PARAMETERS OF THE MINIMUM RUN
THE INITIAL CONDITIONS
  1.000     0.730    1.185
  0.995     0.627    0.364

MASS RATIOS:
PAYLOAD:                0.100
NUCLEAR PROPULSION: 0.166
POWER GENERATION:   0.125
ION PROPELLANT:     0.250
RP1MIN= 0.299    RP1MAX= 0.340
BETA (THRUST/WT RATIO)=  3.000
SP. POWER ALPHA (M*M/S*S*S)= 0.1000
OVERALL MINTIME (DAYS)=  129.00
PERCENT OF EXCESS TO DV1=  70.00
EXCESS FUEL RATIO=  0.415E-01
INJECTION ANGLE (DEG)= -10.00
THE PARAMETERS OF THE MINIMUM RUN
THE INITIAL CONDITIONS
  1.00      0.280E-01   1.112
  1.228     0.769       0.807
```

BIBLIOGRAPHY:

1. Ivanov, A.A., et al. "Concept of Nuclear Power-and -Propulsion System for a Manned Mission to Mars." Sixth Symposium Space Nuclear Power Systems, Albuquerque, New Mexico, January 1989.

2. Bryson, Arthur E. and Yu-Chi Ho. Applied Optimal Control. Waltham, Massachusetts: Blaisdell Publishing Company, 1969.

3. Angelo, Joseph A. and David Buden. [ed.] Space Nuclear Power. Malabar, Florida: Orbit Book Company, 1985.

4. Ley, Willy and Wernher Von Braun. The Exploration of Mars. New York: The Viking Press, 1956.

5. Archer, J.L. and Lankford Cain, Jr. Mars Round Trip Parametric Data For Various Staytimes (1982 and 1986 Oppositions). Boeing Company Space Division Launch Systems Branch, Huntsville Alabama, D5-13489. February 28, 1969.

6. Yoshimura, Shoichi, et al. Optimal Low-thrust Interplanetary Transfer Including Earth Escape Spiral Trajectory

7. Miles, Frank and Nicholas Booth. Race to Mars. New York: Harper & Row Publishers, 1988.

8. Stuhlinger, Ernst. Ion Propulsion for Space Flight. New York: McGraw-Hill, 1964.

9. Stuhlinger, Ernst , et al. "Study of a Nerva-Electric Manned Mars Vehicle." AIAA/AAS Stepping Stones to Mars Meeting. Baltimore, Maryland: American Institute of Aeronautics and Astronautics, March 1966.

10. Wiesel, W.E. Advanced Astrodynamics. Unpublished work, November 1987.

11. Moyer, H. Gardner, and Gordon Pinkham. "Several Trajectory Optimization Techniques, Part II: Application." from Computing Methods in Optimization Problems. New York: Academic Press, 1964.

12. Bate, Roger R., Donald D. Mueller, and Jerry E. White. Fundamentals of Astrodynamics. New York: Dover Publications, 1971.

13. Willis, Edward A. Jr. Comparison of Trajectory Profiles and Nuclear-Propulsion-Module Arrangements for Manned Mars and Mars-Venus Missions. Published report, NASA TN D-6176. Cleveland,Ohio: Lewis Research Center, February 1971.

14. Jahn, Robert G. Physics of Electric Propulsion. New York: McGraw-Hill Book Company, 1968.

15. Sutton, George P. Rocket Propulsion Elements. New York: John Wiley & Sons, Inc., 1986.

16. Beveridge, John H. "Feasibility of Using a Nuclear Rocket Engine for Electrical Power Generation", in AIAA/SAE 7th Propulsion Joint Specialist Conference. Salt Lake City, Utah: American Institute of Aeronautics and Astronautics, June 1971.

17. Lee, A.W. Mars Stopover: Parametric Performance Requirements at Earth and Mars. Boeing Company Space Division Launch Systems Branch, Huntsville Alabama, D5-13414. January 18, 1968.

18. Cantwell, J.R. Performance of Planetary Mission Vehicles with Common Nuclear Propulsion Modules. Boeing Company Space Division Launch Systems Branch, Huntsville Alabama, D5-13437. March 20, 1968.

19. Kenneth, Paul and Gerald E. Taylor. Computation of Optimal Interplanetary Low-thrust Trajectories with Bounded Thrust Magnitude by Means of the Generalized Newton-Raphson Method. Grumman Research Department, Bethpage New York RM-273J. April 19, 1965.

20. Melbourne, W.G., D.E. Richardson, and C.G. Sauer. Interplanetary Trajectory Optimization with Power-Limited Propulsion Systems. Technical Report No. 32-173, Jet Propulsion Laboratory, California Institute of Technology Pasadena, California February 26, 1962.

21. Krzywoblocki, M.Z. A Study of Special Interplanetary Flight Problems. DA-11-022-ORD-2835 University of Illinois Urbana, Illinois. March 4, 1959.

22. Oliveri, Robert A. Study of a Mars Manned Spacecraft -Space Station Rendezvous and Associated Transfer from Earth. Thesis. Wright-Patterson AFB: Air Force Institute of Technology, March 1968.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GA/ENY/90J-1	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Engineering 6b. OFFICE SYMBOL (If applicable) AFIT/EN	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Air Force Institute of Technology Wright-Patterson AFB OH 45433-6583	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO | WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)

TRAJECTORY OPTIMIZATION OF A BIMODAL NUCLEAR POWERED SPACECRAFT TO MARS

12. PERSONAL AUTHOR(S)
Steven R. Oleson, B.S., GS-12, USAF

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Thesis	FROM ______ TO ______	1990 May 29	172

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
03	03		Celestial Mechanics

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Thesis Advisor: Dr. William E. Wiesel
Professor of Astronautical Engineering
Department of Aeronautics and Astronautics

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. William E. Wiesel, Professor	22b. TELEPHONE (Include Area Code) (513) 255-9915 22c. OFFICE SYMBOL ENY

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

Abstract

Minimum flight times for a bimodal, nuclear powered spacecraft are sought. A direct trajectory from Earth to Mars is utilized. Earth escape and Mars braking is accomplished with a high thrust, nuclear thermal propulsion unit, while the interplanetary transit is achieved by a low thrust, electric propulsion unit whose thrusting direction may be varied. An existing method that maximizes circular orbit transfer is adapted to the problem by simplifying the escape and braking conditions and requiring the final orbit to be that of Mars thus obtaining minimum flight times. Low thrust direction history, $\tilde{\Phi}(t)$, excess high thrust fuel division between the escape and braking burns, and optimal escape injection angles are found that determine the minimum flight time. Finally, the size of the low thrust propulsion is also varied to find the minimum time of flight.